Strictness of Rate-Latency Service Curves

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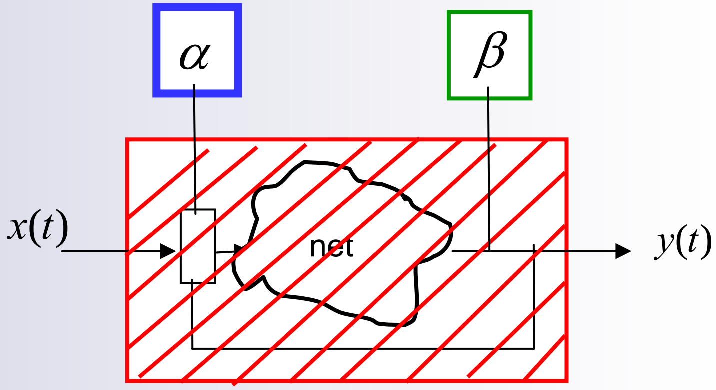
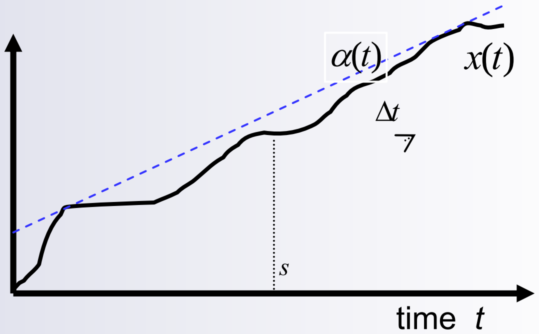
Network Calculus



- Deterministic modelling of traffic flows

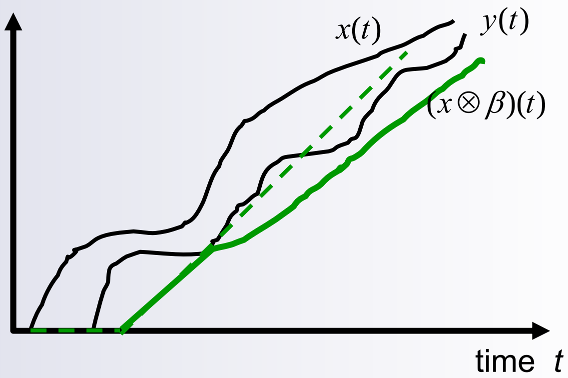
  - Arrival curve α
    x is constrained by $\alpha \Leftrightarrow$ for all Δt :

$$x(s+\Delta t) - x(s) \leq \alpha(\Delta t)$$

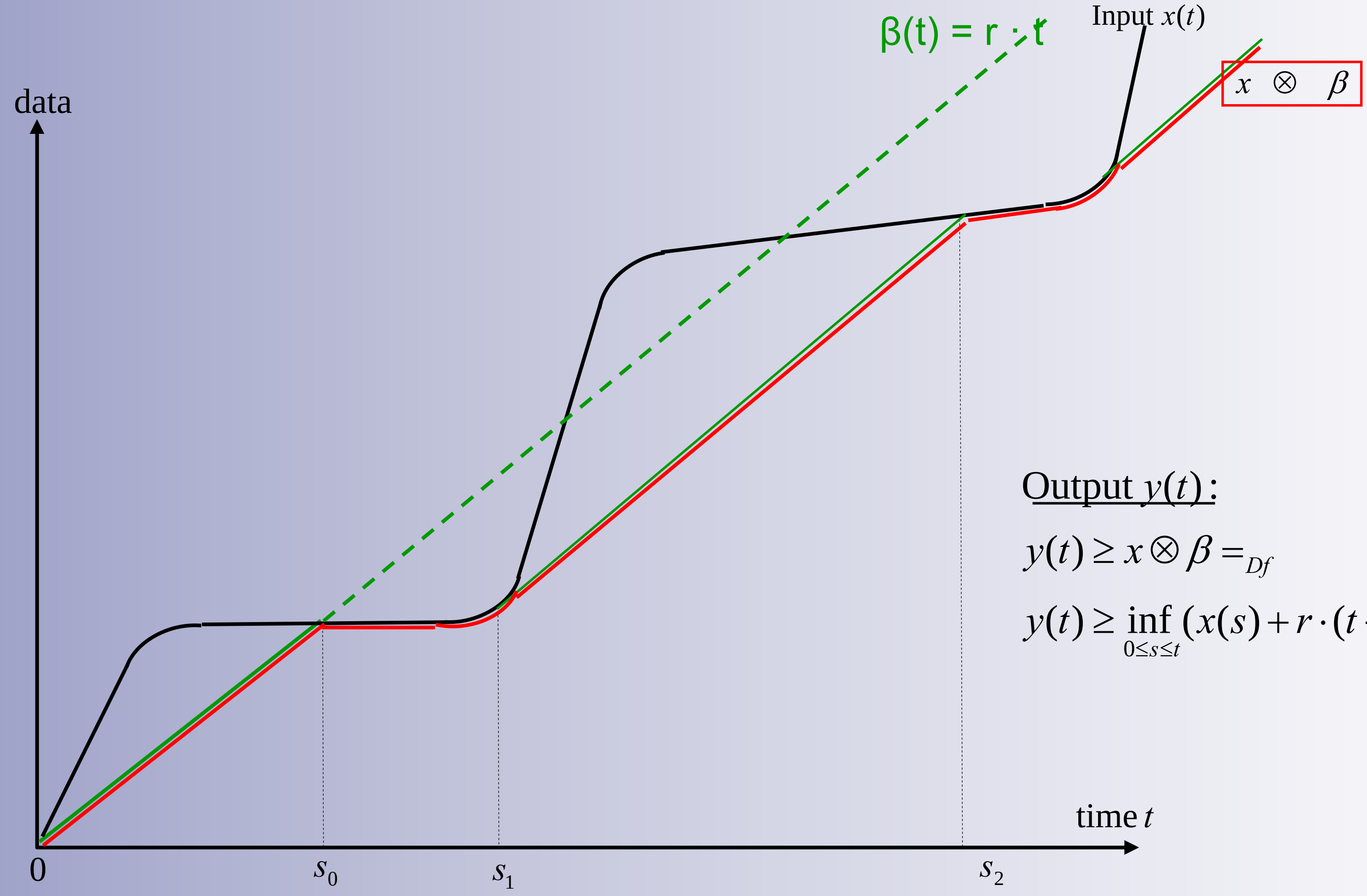


  - Service curve β
    y is low-bounded by $(x \otimes \beta)(t)$:

$$y(t) \geq \inf_{s \leq t} (x(s) + \beta(t-s))$$



Modelling of traffic flows



Output $y(t)$:

$$y(t) \geq x \otimes \beta =_{Df}$$

$$y(t) \geq \inf_{0 \leq s \leq t} (x(s) + r \cdot (t - s))$$

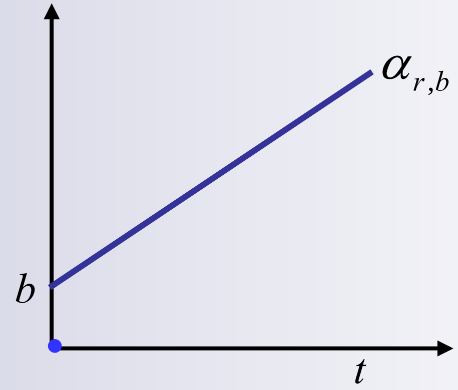
$$\left.\begin{array}{llll} 0 \leq t \leq s_0 : & \inf \text{ at } s = 0 & \rightarrow y \geq x \otimes \beta = r \cdot t \\ s_0 \leq t \leq s_1 : & \inf \text{ at } s = t & \rightarrow y \geq x \otimes \beta = x(t) \\ s_1 \leq t \leq s_2 : & \inf \text{ at } s = s_1 & \rightarrow y \geq x \otimes \beta = x(s_1) + r \cdot (t - s_1) \\ & \vdots & \end{array}\right\} y(t) \geq r \cdot t \wedge x(t) \wedge (x(s_1) + r \cdot (t - s_1)) \wedge \cdots$$

Modelling of traffic flows

- Example

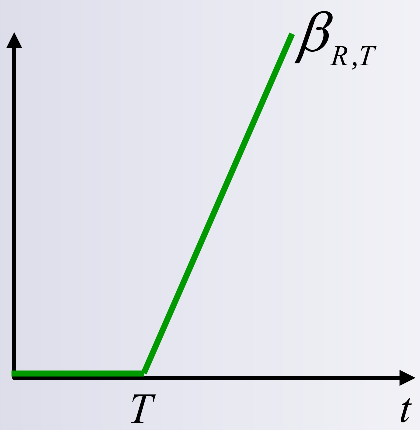
  - Arrival curve: Token Bucket

$$\alpha_{r,b}(t) = \begin{cases} r \cdot t + b & \text{t} > 0 \\ 0 & \text{t} = 0 \end{cases}$$



  - Service curve: Rate-Latency

$$\beta_{R,T}(t) = \begin{cases} \mathrm{R} \cdot (\mathrm{t} - \mathrm{T}) & t > T \\ 0 & t \leq T \end{cases}$$



$\beta_{R,T}$: An incoming input is served with minimum rate R after a possibly maximal delay T (worst case scenario)

Bounds of Backlog, Delay and Output

- Theorem: **Three Bounds**
  - Backlog bound *v*

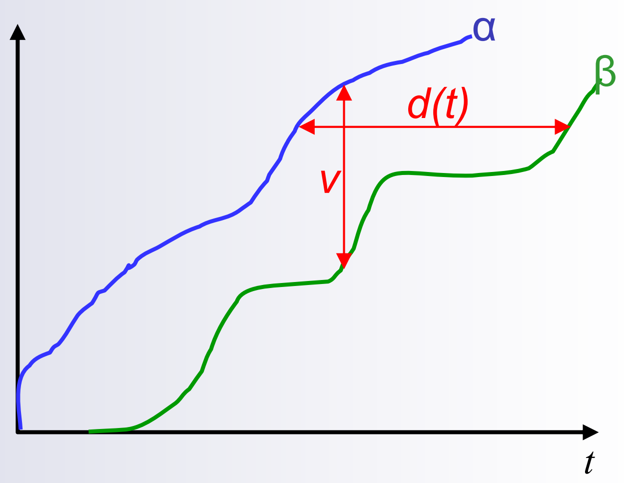
$$v(t) := x(t) - y(t) \le \sup_{s\ge 0}\{\alpha(s) - \beta(s)\}$$

  - Delay bound *d(t)*

$$d(t) \le h(\alpha,\beta) := \sup_{t\ge 0}\{\inf(\tau \ge 0 : \alpha(t) \le \beta(t+\tau))\}$$

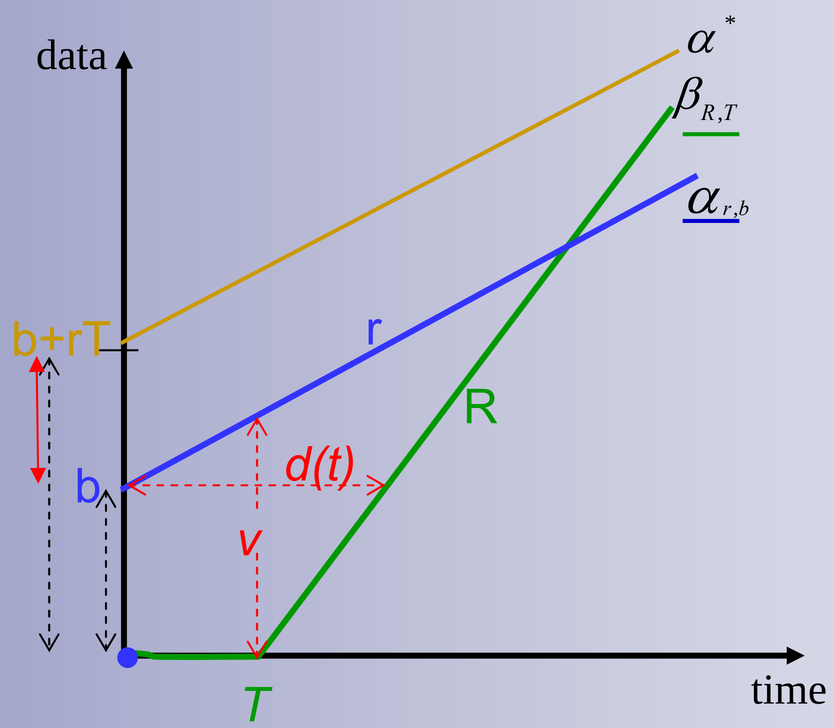
  - Output bound α^*

$$y(t) - y(s) \le \alpha^* = \sup_{s\ge 0}\{\alpha(t+s) - \beta(s)\}$$



Bounds of Backlog, Delay and Output

- Example: Token bucket input $\alpha_{r,b}$
  Rate-latency output $\beta_{R,T}$



Output Bound α^* :

$$(\alpha^* = rt + \underbrace{b + rT}_{\text{new burst}} = r(t+T) + b$$

Backlog Bound :

$$v(t) = b + rT$$

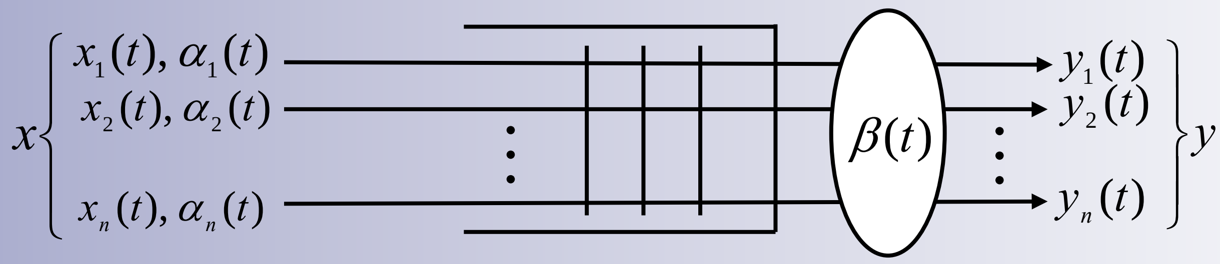
Delay Bound :

$$d(t) \leq T + b/R$$

$$delay \leq latency + \frac{burst}{service\ rate}$$

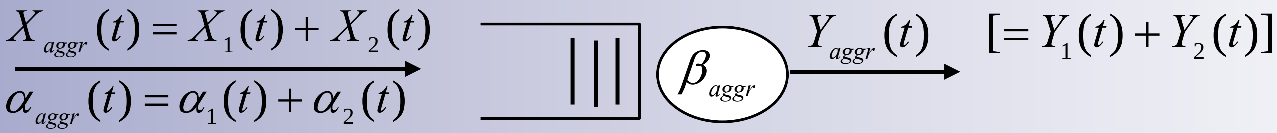
Aggregate scheduling

- Aggregated input $x = \sum_i x_i$
- Aggregated output $y = \sum_i y_i$
- Aggregated arrival curve $\alpha = \sum_i \alpha_i$
- Total Service curve $\beta(t) = \beta_{aggr}(t)$



Multiplexing

- Single flow worst-case analysis based on $\beta(t) = \beta_{aggr}(t)$



<u>Demultiplexing:</u>

- Flow 1 and flow 2 interfere with each other – how much service e.g. is left for flow 1?

- What is maximum delay $d_1(t)$ e.g. of flow X_1 after servicing X_{aggr} and demultiplexing ?
  $\Rightarrow$

- Does service curve β_1 exist for single service X_1 with $Y_1 \geq X_1 \otimes \beta_1$

  such that: $d_1(t) \leq \sup_{t \geq 0}\{\inf(\tau \geq 0 : \alpha_1(t) \leq \beta_1(t+\tau))\}$?

Multiplexing

- Theorem: Aggr. Service Curve minus flow 2-Arrival Curve

$$\beta_{1,\tau}(t) = [\beta_{aggr}(t) - \alpha_2(t-\tau)]^+ \cdot 1_{t>0}$$

- Is generally true for **FIFO** multiplexing

$$\beta_1(t) = [\beta_{aggr}(t) - \alpha_2(t)]^+ \cdot 1_{t>0}$$

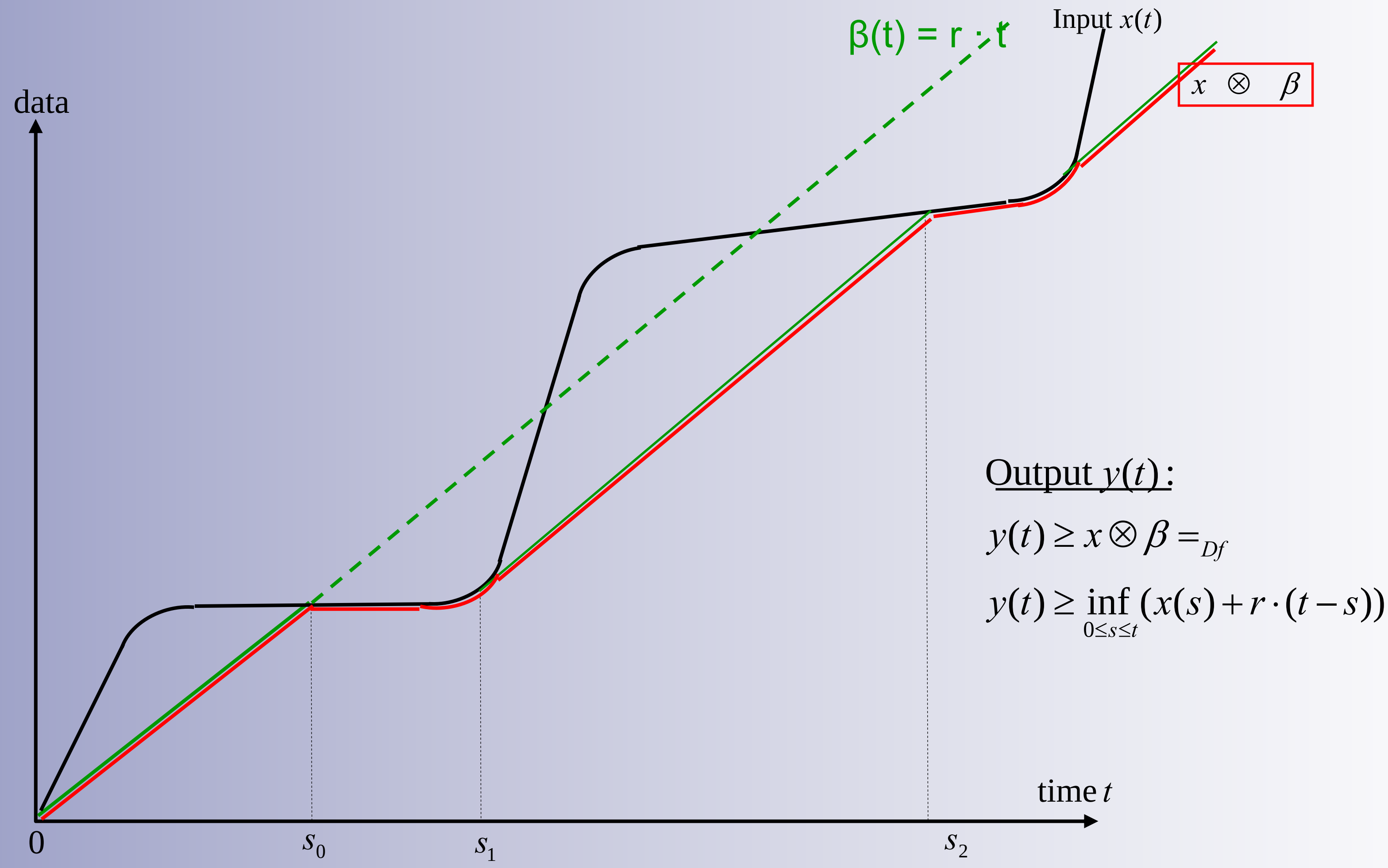
- Is true for **Blind** multiplexing only if **service curve $\beta_{aggr}(t)$** is **strict**

(Blind: Service of flow x_1 and x_2 with unknown arbitration)

- Definition :

Service curve β of a system S is a strict service curve if during any backlogged period $u = [s,t]$ the output y is at least equal to $\beta(u)$:
$y(t) - y(s) \geq \beta(s-t)$.

Strictnes of service curves



Backlogged period $[0, s_0]$: $\rightarrow y(s_0) - y(0) \geq \beta(s_0 - 0) = r \cdot s_0$

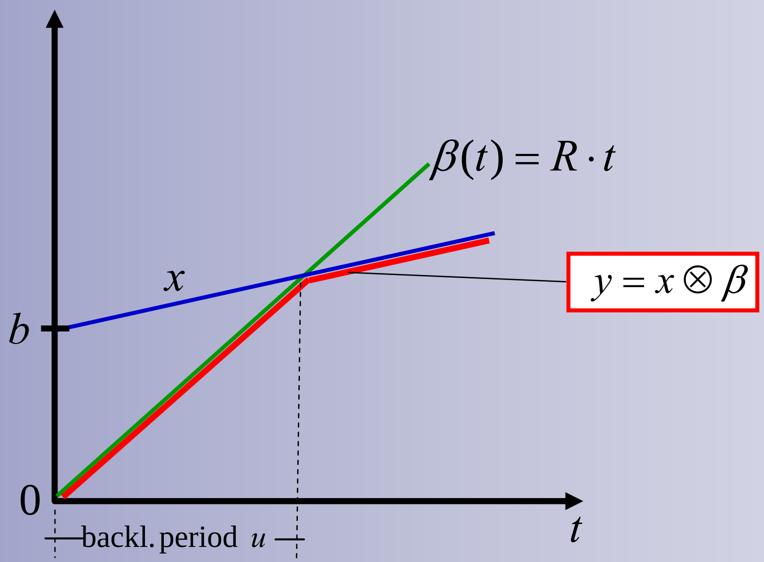
Backlogged period $[s_1, s_2]$: $\rightarrow y(s_2) - y(s_1) \geq \beta(s_2 - s_1) = r \cdot (s_2 - s_1)$

Strictnes of service curves

Example: input $x(t) = r \cdot t + b$

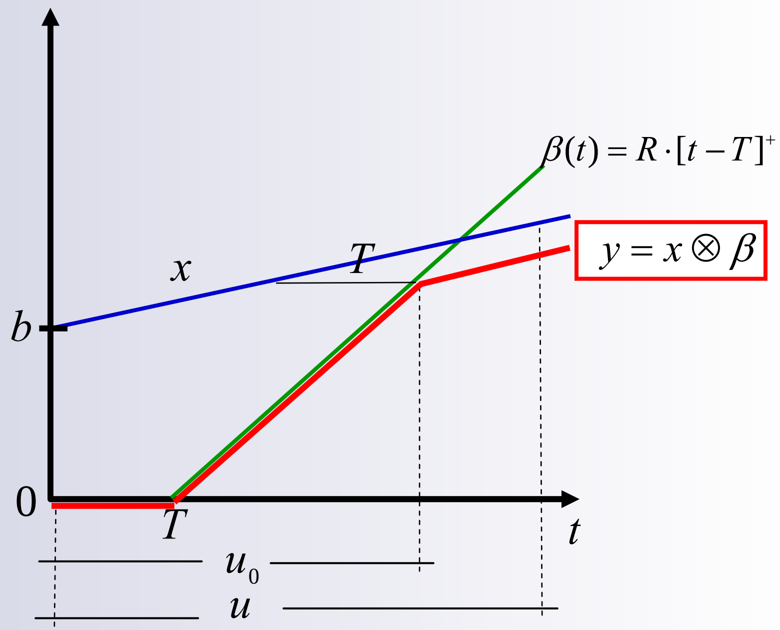
$$\text{output} = y(t) \geq x \otimes \beta = (r \cdot t + b) \otimes \beta_{R,T} = \{b + r(t - T)\} \wedge \{R(t - T)\}$$

Strict



$$y(u) - y(0) \geq \beta(u)$$

Non-strict



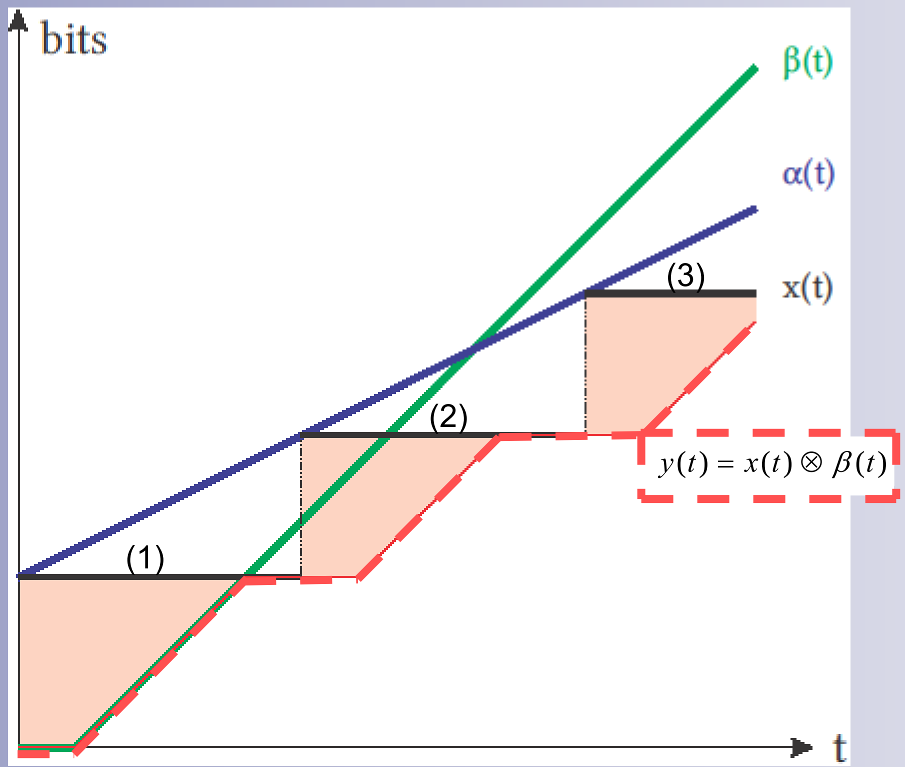
$(x \otimes \beta_{R,T})(u_0) \geq \beta_{R,T}(u_0)$ but $(x \otimes \beta_{R,T})(u) < \beta_{R,T}(u)$

$\Rightarrow y(u) - y(0) \not\geq \beta(u)$!

(here: busy period is never ending)

Strictnes of service curves

- Another example



- service curve $\beta = \beta_{R,T}$
- arrival curve $\alpha = \alpha_{r,b}$
- input stair function x
- output function y

$\Rightarrow$

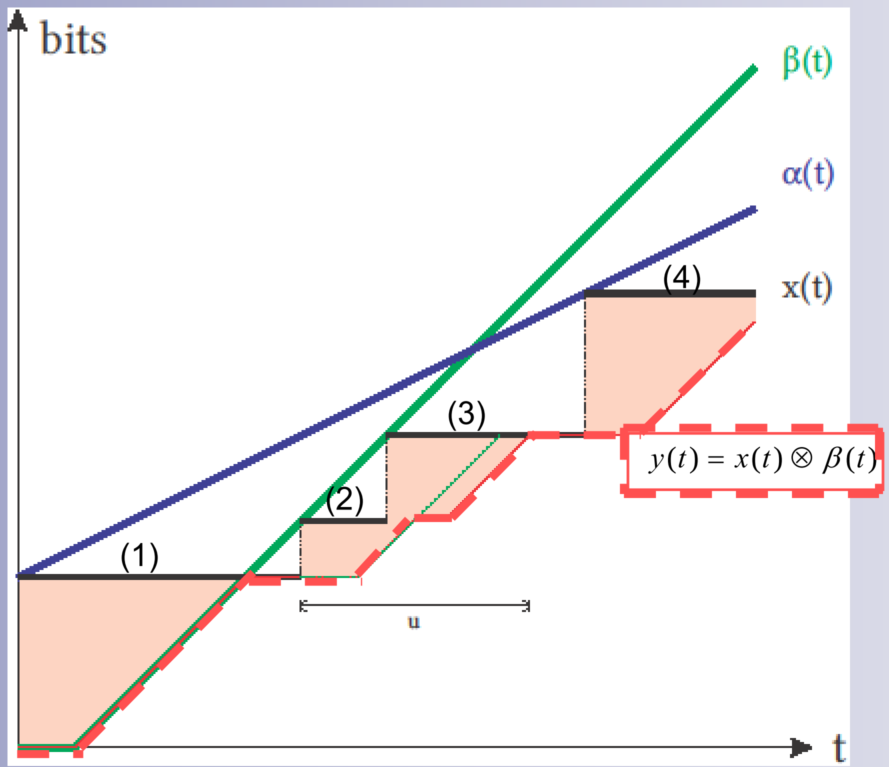
β is strict

Notice:

Each input of x starts a new backlogged period

Strictnes of service curves

- Non-strict



- service curve $\beta = \beta_{R,T}$
- arrival curve $\alpha = \alpha_{r,b}$
- input stair function x
- output function y

$\Rightarrow$

β is non-strict

Notice:

Input (3) of *x* starts inside the backlogged period u

Strictnes of service curves

Remark:

- Any strict service curve is a service curve
- being strict or non-strict depends on service curve **and** input *x* (in each case verifying is required)
- hope: at least in case of token bucket similar input functions with rate-latency service curves decision is easier ⇨

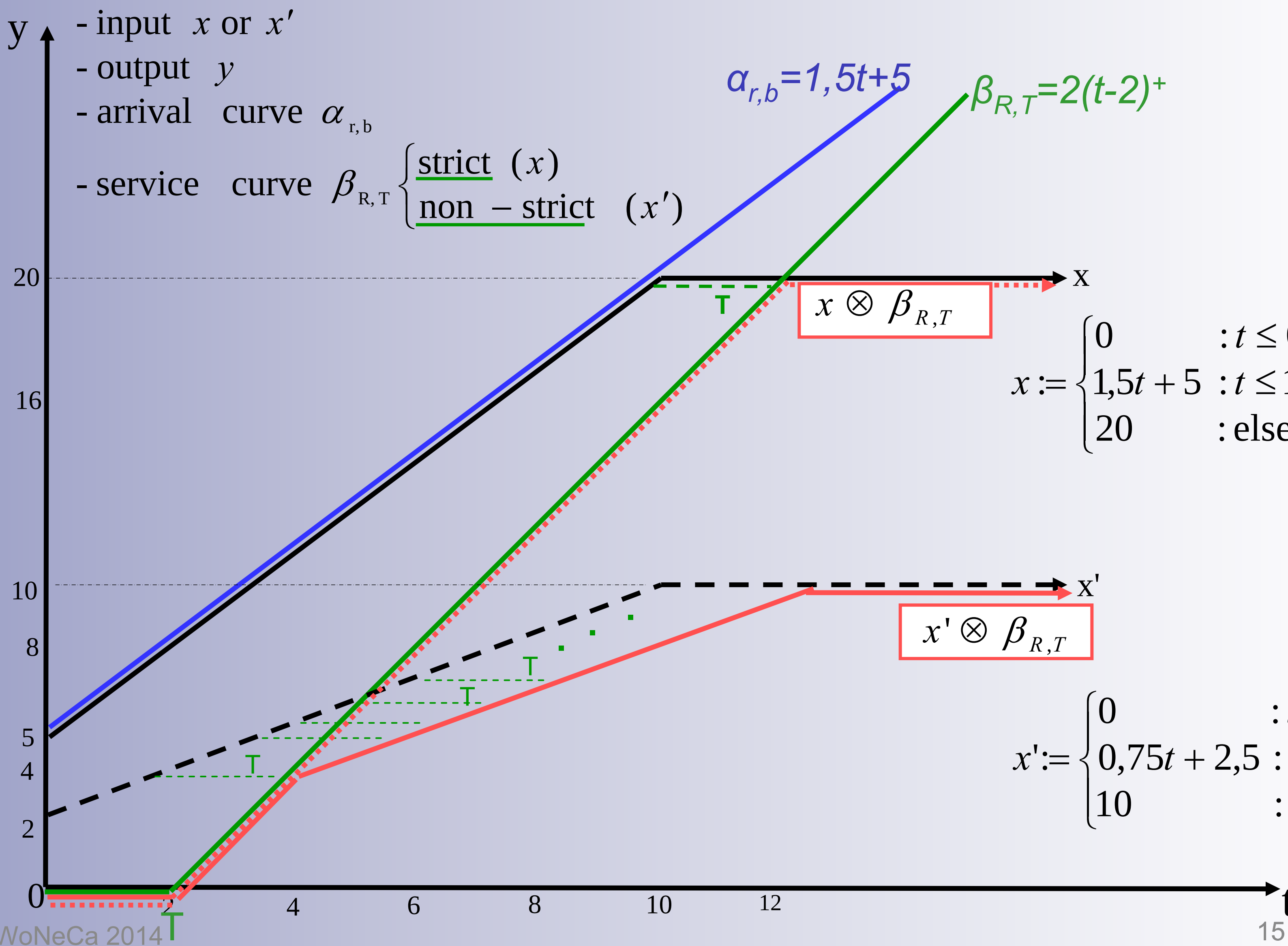
- Theorem:

  Given a sytem with rate - latency service curve $\beta_{R,T}$ (worst case scenario) and token bucket arrival curve $\alpha_{r,b}$ with r < R and T > 0. $\beta_{R,T}$ can not be strict, if the input $x(t)$ is a strictly increasing function.

  $[f \text{ strictly increasing } \Leftrightarrow \forall a, b \text{ with } a < b : f(a) < f(b)]$

  Remark:
  Unfortunately, if input *x* is not strictly increasing - one can not follow that $\beta_{R,T}$ is strict

Examples of strictnes & non-strictness



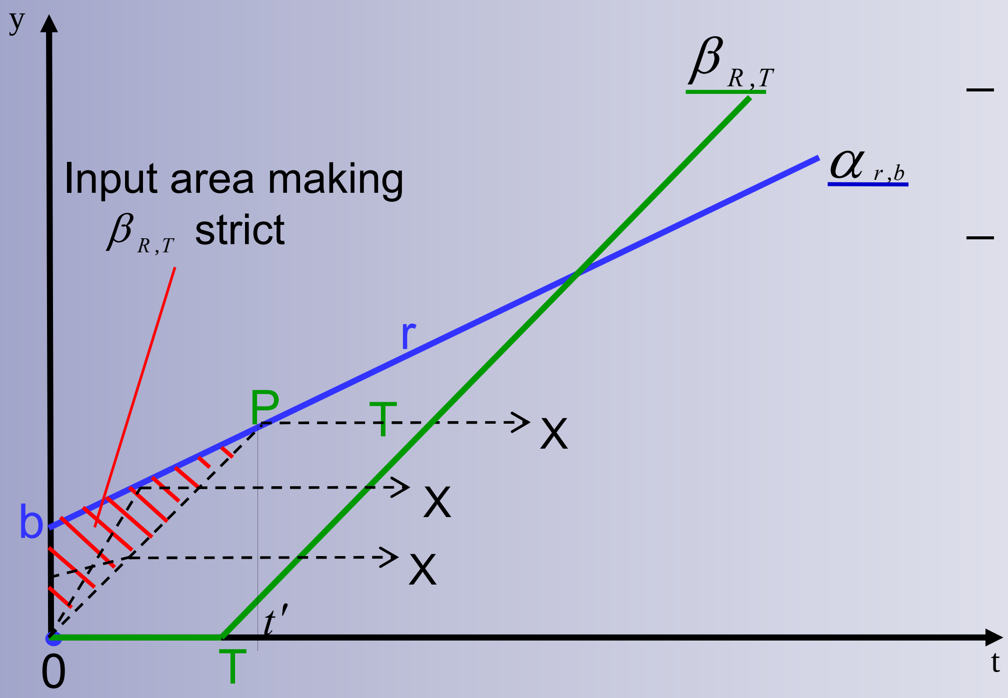
$$x := \begin{cases} 0 & : t \le 0 \\ 1{,}5t + 5 & : t \le 10) \\ 20 & : \text{else} \end{cases}$$

$$x' := \begin{cases} 0 & : t \le 0 \\ 0{,}75t + 2{,}5 & : t \le 10 \\ 10 & : \text{else} \end{cases}$$

Examples

- Obviously, all input functions x causes a strict service curve $\beta_{R,T}$ with

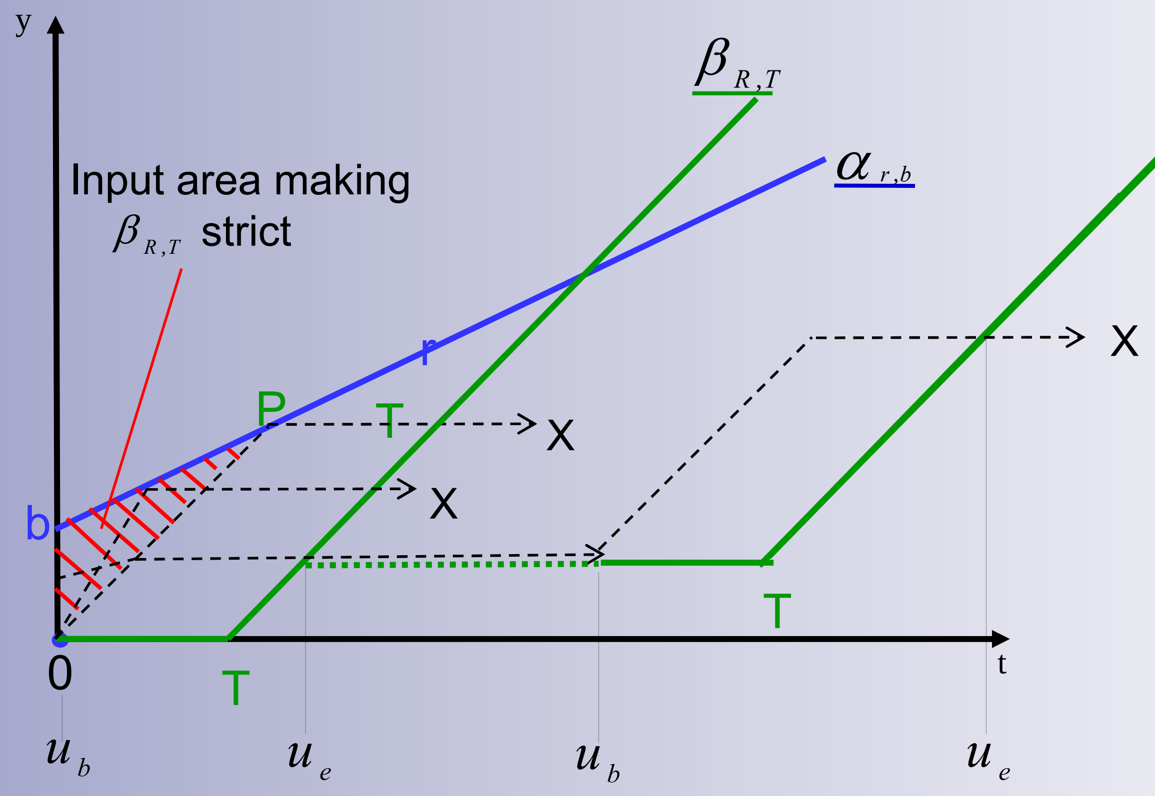
$$x := \begin{cases} m t + n & t: \ t \le t' \\ \text{constant} & : \ \text{else} \end{cases}$$



- constant part of x starts within or at brink of triangle 0bP
- P is intersection of $\alpha_{r,b}$ with curve y=R·t

⋯ or multiple pattern of this

Examples



u_e – end of backlogged period u

u_b – begin of backlogged period u

Example: Service curve for left-over flow

Theorem (Blind):

$$\beta_1(t) = [\beta_{aggr}(t) - \alpha_2(t)]^+ \cdot 1_{t>0}$$

Is true for Blind multiplexing only if service curve $\beta_{aggr}(t)$ is **strict** !

Example:

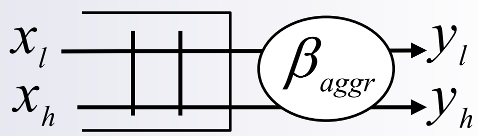
Blind – complete unknown arbitration between two flows ⇨

worst case for flow 1: preemptive priority schedule in favour of flow 2:

flow 1 = ‘low‘ $:= x_l$ flow 2 = ‘high‘ $:= x_h$

$$x_l := \begin{cases} 0 & : t \le 0 \\ 0{,}5t + 2 & : t \le 10 \\ 7 & : \text{else} \end{cases} \qquad x_h := \begin{cases} 0 & : t \le 0 \\ t + 3 & : t \le 10 \\ 13 & : \text{else} \end{cases}$$

$$\Rightarrow \quad \text{the aggregated input } x = x_l + x_h = \begin{cases} 0 & : t \le 0 \\ 1{,}5t + 5 & : t \le 10 \\ 20 & : \text{else} \end{cases}$$



Example: Service curve for left-over flow

Again: - Arrival curve $\alpha_{r,b} := 1{,}5t + 5$ if $t > 0$, zero else

- Aggregated service curve $\beta_{aggr} = \beta_{R,T} := 2(t-2)^+$

$\Rightarrow \quad \beta_{aggr}$ (in connection with aggregated input $x = x_l + x_h$) is strict.

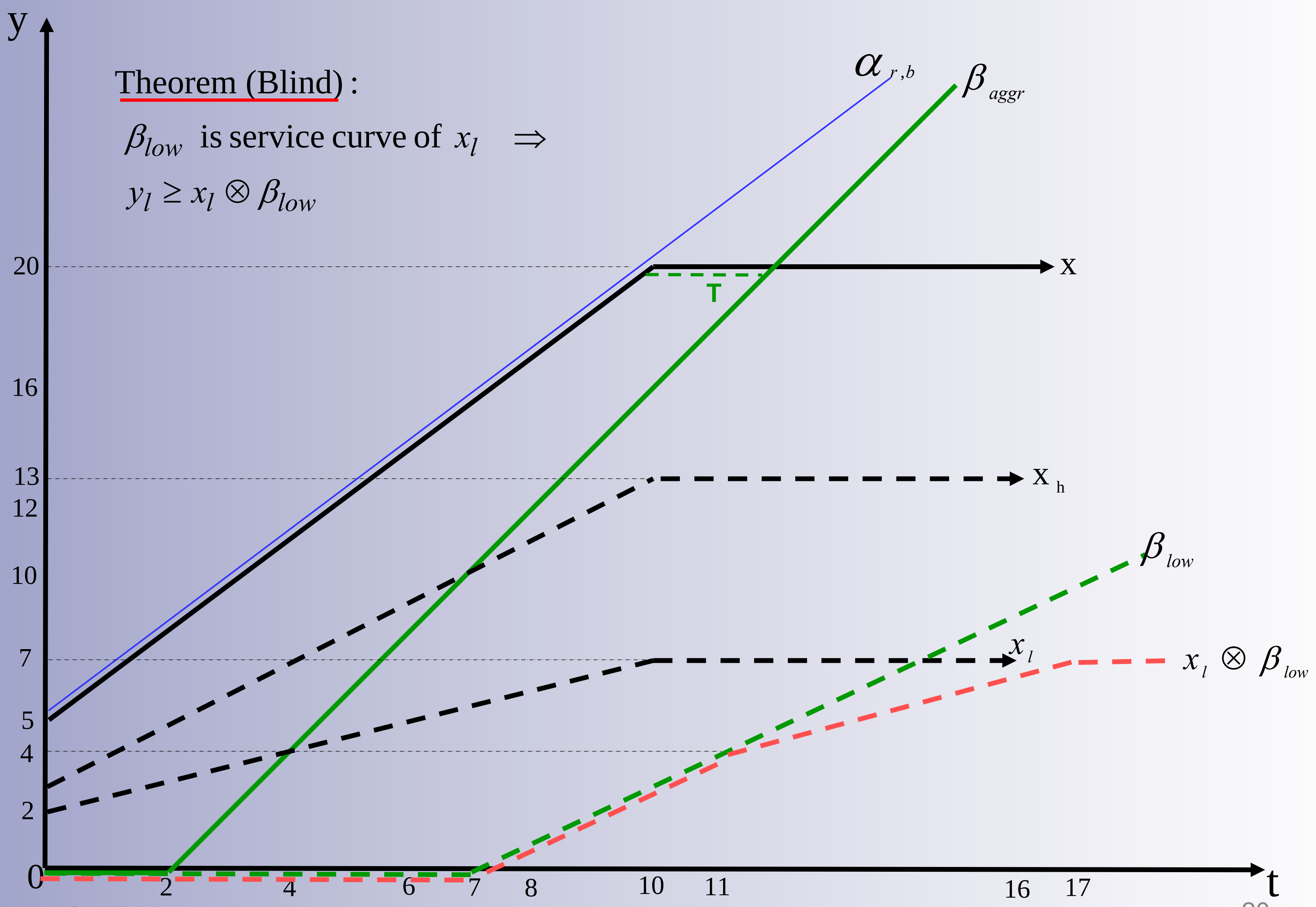
Be arrival curve of x_h : $\alpha_h = t + 3$ As per Theorem (Blind) $\Rightarrow$

$\beta_{low}(t) = [\beta_{aggr}(t) - \alpha_h(t)]^+ = [2(t-2) - (t+3)]^+ = [t-7]^+$ is service curve of x_l.

$$\Rightarrow \text{ for any low-priority output } y_l: \quad y_l \geq (x_l \otimes \beta_l)(t) = \begin{cases} (t-7)^+ & : t \leq 11 \\ 2 + 0{,}5(t-7)^+ & : 11 < t \leq 17 \\ 7 & : \text{else} \end{cases}$$

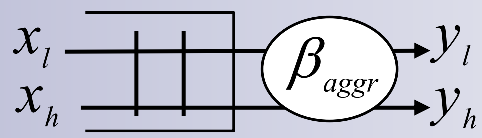
$\Rightarrow$ y_l is lower bounded by $(t-7)^+$ if $t \in [0, 11]$
by $2 + 0{,}5\,(t-7)$ if $t \in [11, 17]$
and by 7 if $t \geq 17$.

Example: Service curve for left-over flow



Aggregate scheduling

- Remark:



Theorem (Blind) formulates sufficient conditions only :

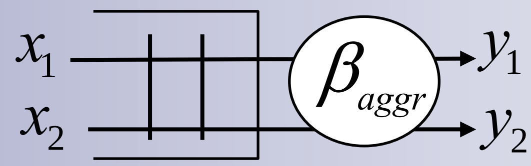
if β_{aggr} is non - strict $\Rightarrow \beta_{\text{low}}(\text{t}) := [\beta_{aggr}(t) - \alpha_h(t)]^+$ can be both, a service curve for flow x_l or not - depending on x_l

- Therefore :

Can we bypass the question of strictness or non - strictness in case of aggregate scheduling ?

Aggregate scheduling

- Trials to bypass question of strictness or non-strictness in case of aggregate scheduling

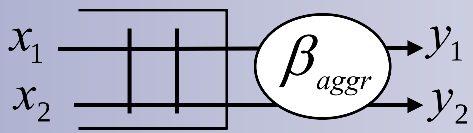


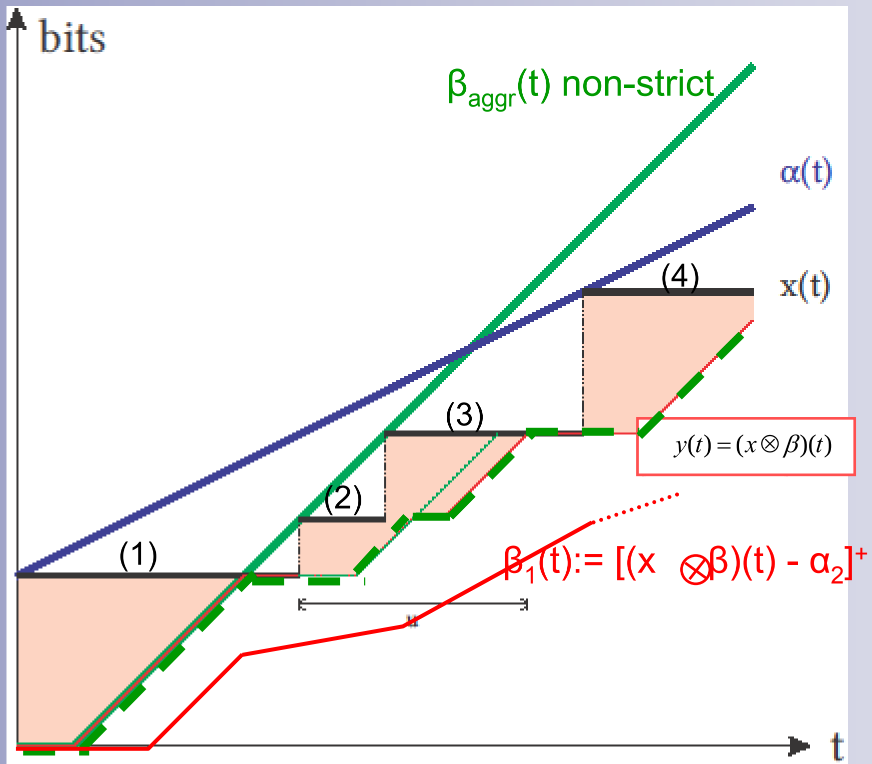
Statement 1:

Given a node serving the flows x_1 and x_2 again with unknown arbitration between the flows, $x = x_1 + x_2$ the aggregated input, and $y = y_1 + y_2$ the aggregated output, β_{aggr} - the service curve of x and flow x_2 is bounded by $K > 0$.

If $\beta_1(t) := [\beta_{aggr}(t) - K]^+$ wide-sense increasing, then β_1 is a service curve for x_1.

Aggregate scheduling





Statement 2:

β_{aggr} is not strict, so you can't reason

$\beta_1(t) := [(\beta_{aggr} - \alpha_2)(t)]^+$ is a service curve for x_1 but

$\Downarrow$

$$\beta_1(t) := [(x \otimes \beta_{aggr})(t) - \alpha_2]^+$$

is a service curve for x_1 !

$x = x_1 + x_2,\ \alpha = \alpha_1 + \alpha_2$
x_2 is α_2 − smooth

Conclusion

- Network Calculus – QoS performance evaluation tool of aggregate multiplexing flows

- Aggregate FIFO and Blind service
  - Strict & non-strict service
  - Strictness - sufficient condition for service curves of single individual flows within blind scheduling

- Strictness or non-strictness – often not a unique feature of service curve per se

- Thanks for your attention !