Window Flow Control Systems with Random Service

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Feedback system

- Feedback system:

For the analysis we use network calculus methodology

- Network calculus has analyzed feedback systems under deterministic assumptions

Open problem in network calculus
Analysis of feedback systems with probabilistic assumptions
Related work

- Performance bonds for flow control protocols
  - Deterministic analysis
  - Min-plus algebra
  - Window flow control model

- A min,+ system theory for constrained traffic regulation and dynamic service guarantees
  - Deterministic analysis
  - Min-plus algebra
  - Window flow control model

- TCP is max-plus linear
  - Deterministic service process
  - Max-plus algebra
  - TCP Tahoe and TCP Reno

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Related work

TCP congestion avoidance\(^4\)
- Deterministic analysis
- Min-plus algebra
- Window flow control model
- TCP Vegas and Fast TCP

Window flow control in stochastic network calculus\(^5\)
- Stochastic analysis
- Min-plus algebra
- Window flow control model


Bivariate network calculus

\[(f \land g)(s, t) = \min\{f(s, t), g(s, t)\}\]
\[(f \otimes g)(s, t) = \min_{s \leq \tau \leq t}\{f(s, \tau) + g(\tau, t)\}\]
\[(f \otimes g)(s, t) \neq (g \otimes f)(s, t)\]

- \((\land, \otimes)\) operations form a non-commutative dioid over non-negative non-decreasing bivariate functions

- discrete-time domain \((t = 0, 1, 2, \ldots)\)

- Sub-additive closure:

\[f^* \triangleq \delta \land f \land f^{(2)} \land f^{(3)} \land \ldots = \bigwedge_{n=0}^{\infty} f^{(n)}\]

where \(f^{(n+1)} = f^{(n)} \otimes f\) for \(n \geq 1\), \(f^{(0)} = \delta\), and \(f^{(1)} = f\)
Moment-generating function network calculus\(^6\)

- Moment-generating function of a random variable \(X\):
  \[
  M_X(\theta) = E[e^{\theta X}]
  \]

- Moment-generating function of operations \(\otimes\) and \(\ominus\):
  \[
  M_{f \otimes g}(-\theta, s, t) \leq \sum_{\tau=s}^{t} M_f(-\theta, s, \tau) M_g(-\theta, \tau, t) \\
  M_{f \ominus g}(\theta, s, t) \leq \sum_{\tau=0}^{s} M_f(\theta, \tau, t) M_g(-\theta, \tau, s)
  \]

- For \(Pr\left(S(s, t) \leq S^\varepsilon(s, t)\right) \leq \varepsilon\), statistical service bound
  \[
  S^\varepsilon(s, t) = \max_{\theta > 0} \frac{1}{\theta} \left\{ \log \varepsilon - \log M_S(-\theta, s, t) \right\}
  \]

State-of-the-art: Window flow control

\[ A' = \min \{ A, D' \} \]

\[ \delta^+ w(s, t) = \begin{cases} w & s \geq t, \\ \infty & s < t \end{cases} \]

\[ D' = D \otimes \delta^+ w = D + w \]

\[ A' - D = \min \{ A, D + w \} - D \leq D + w - D = w \]
State-of-the-art: Window flow control

- Delay element represent feedback delay:

\[ \delta_d(s, t) = \delta(s, t - d) \]

- Equivalent feedback service:

\[ S_{\text{win}} = \left( S \otimes \delta_d \otimes \delta^+ \right)^* \otimes S \]
Feedback system with $w > 0$, $d \geq 0$ and with an additive service process

$$S(s, t) = \sum_{k=s}^{t-1} c_k$$

$c_k$'s are arbitrary sequence of non-negative random variables

If feedback delay is one ($d = 1$),

$$S_{\text{win}}(s, t) = \sum_{k=s}^{t-1} \min \{c_k, w\}$$
For the equivalent service process $S_{\text{win}}$ of a general feedback system with window size $w > 0$, and feedback delay $d \geq 0$, we have

- **Upper and lower bounds:**

  $$S'_{\text{win}}(s, t) < S_{\text{win}}(s, t) < \min \{ S(s, t), \left\lceil \frac{t-s}{d} \right\rceil w \}$$

  $S'_{\text{win}}(s, t)$ is the equivalent service process of the feedback system with window size $w' = w/d$ and feedback delay $d' = 1$

- The lower bound corresponds to the exact result
Results: Equivalent service

- Feedback system with window size $w > 0$ and delay $d \geq 0$:

\[
S_{\text{win}}(s, t) = \bigwedge_{n=0}^{\left\lfloor \frac{t-s}{d} \right\rfloor} \left\{ \min C_n(s, t) \left( \sum_{i=1}^{n} \left( S(\tau_{i-1}, \tau_{i} - d) \right) + S(\tau_{n}, t) \right) + nw \right\}
\]

where $C_n(s, t)$ is given as

\[
C_n(s, t) = \{ s = \tau_0 \leq \cdots \leq \tau_n \leq t \mid \forall i = 0, \ldots, n \ \tau_i - \tau_{i-1} \geq d \}\]
Variable Bit Rate (VBR) server

\[ S(s, t) = \sum_{k=s}^{t-1} c_k \]

where \( c_k \)'s are independent and identically distributed random variables

For a feedback system with VBR server with window size \( w > 0 \) and delay \( d \geq 0 \):

\[
M_{S_{\text{win}}}(-\theta, s, t) \leq \left( M_c(-\theta)^d + de^{-\theta w} \right) \left\lfloor \frac{t-s}{d} \right\rfloor
\]

\( M_c(\theta) \) is the moment-generating function of \( c_k \),

\[
M_c(\theta) = E \left[ e^{\theta c_k} \right]
\]
Markov-modulated On-Off (MMOO) server operates in two states:

- **ON** (state 1): The server transmits a constant amount of \( P > 0 \) units of traffic per time slot, \( c_k = P \)
- **OFF** (state 0): The server does not transmit, \( c_k = 0 \)

The MMOO server offers an additive service process:

\[
S(s, t) = \sum_{k=s}^{t-1} c_k
\]

For a feedback system with MMOO server with window size \( w > 0 \) and delay \( d \geq 0 \), if \( p_{01} + p_{10} < 1 \):

\[
M_{\text{win}}(-\theta, s, t) \leq \left( m_+(-\theta)^d + d e^{-\theta w} \right) \left\lfloor \frac{t-s}{d} \right\rfloor
\]

\( m_+(\theta) \) is the larger eigenvalue of the matrix:

\[
L(\theta) = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{\theta P} \end{pmatrix}
\]
Numerical results: Statistical service bounds

\[ S_{\text{win}}(s, t) = \max_{\theta > 0} \frac{1}{\theta} \left\{ \log \varepsilon - \log M_{S_{\text{win}}(-\theta, s, t)} \right\} \]

VBR server with exponential \( c_k \)

MMOO server with \( p_{00} = 0.2, p_{11} = 0.9, P = 1.125 \text{ Mb} \)

Average rate = 1 Gbps, time unit = 1 ms, \( w/d = 500 \text{ Mbps} \), \( \varepsilon = 10^{-6} \)
Numerical results: Effective capacity

\[ \gamma_{S_{\text{win}}}(-\theta) = \lim_{t \to \infty} -\frac{1}{\theta t} \log M_{S_{\text{win}}}(-\theta, 0, t) \]

VBR server with exponential \( c_k \)

MMOO server with \( p_{00} = 0.2, \ p_{11} = 0.9, \ P = 1.125 \text{ Mb} \)

Effective capacity (Mbps)

Average rate = 1 Gbps, \( w/d = 500 \text{ Mbps} \)
Numerical results: Backlog and delay bounds

\[ A(s, t) = \sum_{k=s}^{t-1} a_k \] with exponential \( a_k \) and average rate \( \lambda \)

**Backlog bound**

**Delay bound**

Exponential VBR, time unit = 1 ms, feedback delay \( d = 1 \) ms
Conclusions

Results:

- Exact results
- Upper and lower service bounds
- Equivalent service of the feedback system
- Bounds for a feedback system with VBR server
- Bounds for a feedback system with MMOO server
- Backlog and delay bounds
Thank you
Q & A