

Network Calculus Tests – Tree Network Configurations

Version 1.1 (2014-Dec-30)



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General Information

- The network calculus analyses presented in this document were created for the purpose of testing the Disco Deterministic Network Calculator (DiscoDNC)¹ – an open-source deterministic network calculus tool developed by the *Distributed Computer Systems (DISCO) Lab* at the University of Kaiserslautern.
- Naming of the individual network configurations depicts the name of the according functional test for the DiscoDNC.
- The naming scheme used in this document is detailed in `NetworkCalculus_NamingScheme.pdf`.
- Arrival bound computations are equivalent to the `PbooArrivalBound_Output_PerHop.java` class of the DiscoDNC.
- The end-to-end left-over service curve for PBOO arrival bounds can be computed by simply convolving the server-local ones.
- Arrival bounds for `PmooArrivalBound.java` and analyses using them are listed only if results are different to PBOO.

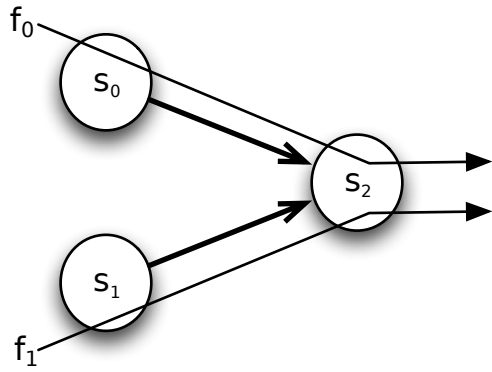
Changelog:

Version 1.1 (2014-Dec-30):

- Streamlined the PMOO left-over latency $T_{e2e}^{l.o.f}$ computation.
- Adapted to naming scheme version 1.1.

¹<http://disco.cs.uni-kl.de/index.php/projects/disco-dnc>

Tree_1SC_2Flows_1AC_2Paths



- $\beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{R_{s_i}, T_{s_i}} = \beta_{20,20}, i \in \{0, 1, 2\}$
- $\mathcal{F} = \{f_0, f_1\}$
- $\alpha^{f_0} = \alpha^{f_1} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25}, n \in \{0, 1\}$

arrivalBound($s_2, \{f_n\}, \mathcal{G}$), $\mathcal{G} = \mathcal{P}(\mathcal{F}) = \alpha_{s_2}^{f_n}, n \in \{0, 1\}$		FIFO_MUX	ARB_MUX
$\alpha_{s_2}^{f_n}$			$= \gamma_{5,25}$
$\alpha_{s_2}^{x f_n}$			$= \gamma_{0,0}$
$\beta_{s_2}^{l.o.f_n} = \beta_{s_2} \ominus \alpha_{s_2}^{x f_n} = \beta_{R_{s_2}^{l.o.f_n}, T_{s_2}^{l.o.f_n}}$	$=$		$= \beta_{20,20}$
$\alpha_{s_2}^{f_n} = \alpha_{s_n}^{f_n} \circlearrowleft \beta_{s_n}^{l.o.f_n} = \gamma_{r_{s_2}^{f_n}, b_{s_2}^{f_n}}$	$r_{s_2}^{f_n}$		$= 5$
	$b_{s_2}^{f_n}$	$\alpha_{s_n}^{f_n}(T_{s_n}^{l.o.f_n}) = 5 \cdot 20 + 25 = 125$	
	$=$		$= \gamma_{5,125}$

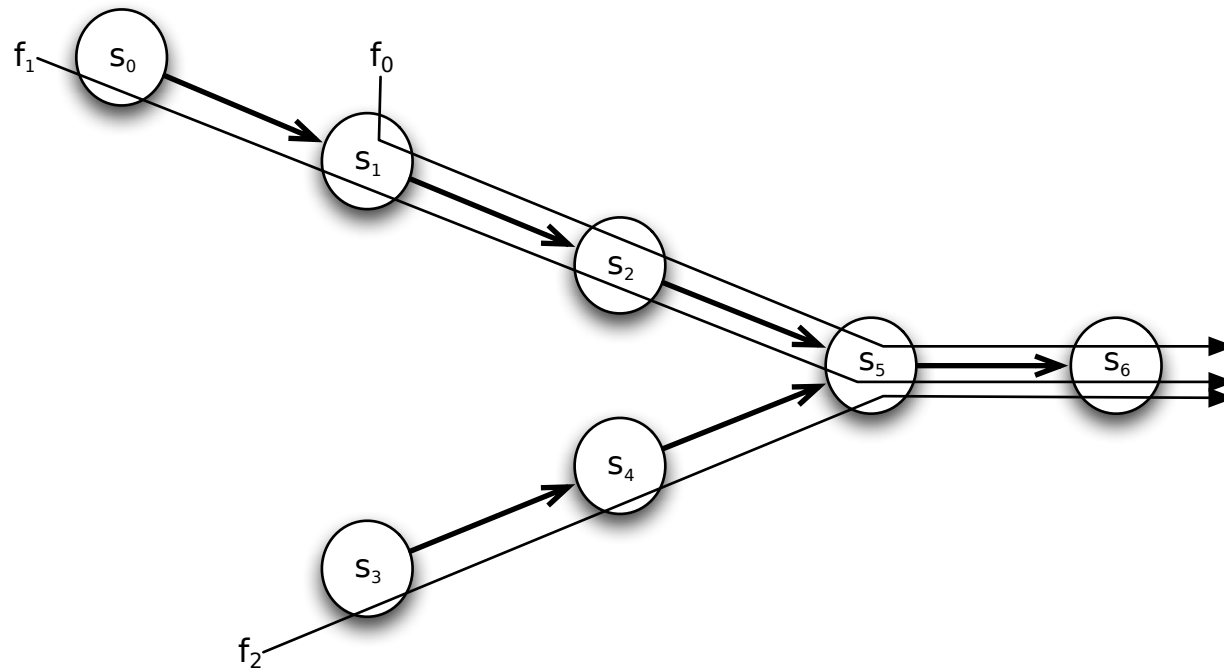
Flow f_n , $n \in \{0,1\}$ (comparable with Tandem_1SC_2Flows_1AC_2Paths)

	TFA	FIFO_MUX	ARB_MUX
s_n	$\alpha_{s_n} = \alpha_{s_n}^{f_n}$	$= \gamma_{5,25}$	
	$D_{s_n}^{f_n}$	$\beta_{s_n} = b_{s_n}^{f_n}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	FIFO per micro flow $\beta_{s_n} = b_{s_n}^{f_n}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$
	$B_{s_n}^{f_n}$	$\alpha_{s_n}(T_{s_n}) = 5 \cdot 20 + 25 = 125$	
s_2	$\alpha_{s_2} = \sum_j \alpha_{s_2}^{f_n}$	$= \gamma_{5,125} + \gamma_{5,125} = \gamma_{10,250}$	
	$D_{s_2}^{f_n}$	$\beta_{s_2} = b_{s_1}$ $20 \cdot [t - 20]^+ = 250$ $t = 32\frac{1}{2}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 250$ $t = 65$
	$B_{s_2}^{f_n}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 250 = 450$	
	$D_{s_i}^{f_n}$	$\sum_{i=\{n,2\}} D_{s_i}^{f_n} = 53\frac{3}{4}$	$\sum_{i=\{n,2\}} D_{s_i}^{f_n} = 86\frac{1}{4}$
	$B_{s_i}^{f_n}$	$\max_{i=\{n,2\}} b_{s_i}^{f_n} = 450$	

	SFA	FIFO_MUX	ARB_MUX
s_n	$\alpha_{s_n}^{x f_n}$	$= \gamma_{0,0}$	
	$\beta_{s_n}^{1.o.f_n} = \beta_{s_n} \ominus \alpha_{s_n}^{x f_n} = \beta_{s_n}$	$= \beta_{20,20}$	
s_2	$\alpha_{s_2}^{x f_n} = \alpha_{s_2}^{f_n}$	$= \gamma_{5,125}$	
	$\beta_{s_2}^{1.o.f_n} = \beta_{s_2} \ominus \alpha_{s_2}^{x f_n} = \beta_{R_{s_2}^{1.o.f_n}, T_{s_2}^{1.o.f_n}}$	$R_{s_2}^{1.o.f_n}$	$[R_{s_2} - r_{s_2}^{x f_n}]^+ = 15$
		$T_{s_2}^{1.o.f_n}$	$\beta_{s_2} = b_{s_2}^{x f_n}$ $20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$
	$=$	$= \beta_{15,26\frac{1}{4}}$	$= \beta_{15,35}$
	$\beta_{e2e}^{1.o.f_n} = \beta_{R_{e2e}^{1.o.f_n}, T_{e2e}^{1.o.f_n}}$	$\otimes_{i=\{n,2\}} \beta_{s_i}^{1.o.f_n} = \beta_{15,46\frac{1}{4}}$	$\otimes_{i=\{n,2\}} \beta_{s_i}^{1.o.f_n} = \beta_{15,55}$
	$D_{s_i}^{f_n}$	$\beta_{e2e}^{1.o.f_n} = b_{s_i}^{f_n}$ $15 \cdot [t - 46\frac{1}{4}]^+ = 25$ $t = 47\frac{11}{12}$	$\beta_{e2e}^{1.o.f_n} = b_{s_i}^{f_n}$ $15 \cdot [t - 55]^+ = 25$ $t = 56\frac{2}{3}$
	$B_{s_i}^{f_n}$	$\alpha_{s_i}^{f_n}(T_{e2e}^{1.o.f_n}) = 5 \cdot 46\frac{1}{4} + 25 = 256\frac{1}{4}$	$\alpha_{s_i}^{f_n}(T_{e2e}^{1.o.f_n}) = 5 \cdot 55 + 25 = 300$

PMOO		ARB_MUX
s_n	$\alpha_{s_n}^{\bar{x}f_n}$	$= \gamma_{0,0}$
	$\alpha_{s_n}^{xf_n}$	$= \gamma_{0,0}$
s_2	$\alpha_{s_2}^{\bar{x}(f_0)}$	$= \gamma_{5,125}$
	$\alpha_{s_2}^{x(f_0)}$	$= \gamma_{5,125}$
$\beta_{e2e}^{l.o.f_n} = \beta_{R_{e2e}^{l.o.f_n}, T_{e2e}^{l.o.f_n}}$	$R_{e2e}^{l.o.f_n} = \bigwedge_{i \in \{n,2\}} (R_{s_i} - r_{s_i}^{xf_n})$	$= (20 - 0) \wedge (20 - 5)$
		$= 15$
	$T_{e2e}^{l.o.f_n} = \sum_{i \in \{n,2\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}f_n} + r_{s_i}^{xf_n} \cdot T_{s_i}}{R_{e2e}^{l.o.f_0}} \right)$	$= 20 + \frac{0 + 0 \cdot 20}{15} + 20 + \frac{125 + 5 \cdot 20}{15}$
		$= 40 + \frac{225}{15}$
	$=$	$= 55$
	$=$	$= \beta_{15,55}$
Df_n		$\beta_{e2e}^{l.o.f_n} = b^{f_n}$
		$15 \cdot [t - 55]^+ = 25$
		$t = 56\frac{2}{3}$
Bf_n		$\alpha^{f_n}(T_{e2e}^{l.o.f_n}) = 5 \cdot 55 + 25$
		$= 300$

Tree_1SC_3Flows_1AC_3Paths



- $\beta_{s_0} = \beta_{s_1} = \beta_{s_2} = \beta_{R_{s_i}, T_{s_i}} = \beta_{20,20}, i \in \{0, 1, 2\}$
- $\mathcal{F} = \{f_0, f_1, f_2\}$
- $\alpha^{f_0} = \alpha^{f_1} = \alpha^{f_2} = \gamma_{r^{f_n}, b^{f_n}} = \gamma_{5,25}, n \in \{0, 1, 2\}$

arrivalBound($s_1, \{f_1\}, \mathcal{G}$), $\mathcal{G} \in \mathcal{P}(\mathcal{F}) = \alpha_{s_1}^{f_1}$		FIFO_MUX	ARB_MUX
$\alpha_{s_0}^{f_1}$			= $\gamma_{5,25}$
$\alpha_{s_0}^{x(f_1)}$			= $\gamma_{0,0}$
$\beta_{s_0}^{1.o.f_1} = \beta_{s_0} \ominus \alpha_{s_0}^{x(f_1)} = \beta_{R_{s_0}^{1.o.f_1}, T_{s_0}^{1.o.f_1}}$	=		= $\beta_{20,20}$
$\alpha_{s_1}^{f_1} = \alpha_{s_0}^{f_1} \circ \beta_{s_0}^{1.o.f_1} = \gamma_{r_{s_1}^{f_1}, b_{s_1}^{f_1}}$	$r_{s_1}^{f_1}$		= 5
	$b_{s_1}^{f_1}$	$\alpha_{s_0}^{f_1}(T_{s_0}^{1.o.f_1}) = 5 \cdot 20 + 25 = 125$	
	=		= $\gamma_{5,125}$

arrivalBound($s_2, \{f_1\}, \{f_0\}) = \alpha_{s_2}^{f_1}$		FIFO_MUX	ARB_MUX
$\alpha_{s_1}^{f_1}$			= $\gamma_{5,125}$
$\alpha_{s_1}^{x(f_1)}$			= $\gamma_{0,0}$
$\beta_{s_1}^{1.o.f_1} = \beta_{s_1} \ominus \alpha_{s_1}^{x(f_1)} = \beta_{R_{s_1}^{1.o.f_1}, T_{s_1}^{1.o.f_1}}$	=		= $\beta_{20,20}$
$\alpha_{s_2}^{f_1} = \alpha_{s_1}^{f_1} \circ \beta_{s_1}^{1.o.f_1} = \gamma_{r_{s_2}^{f_1}, b_{s_2}^{f_1}}$	$r_{s_2}^{f_1}$		= 5
	$b_{s_2}^{f_1}$	$\alpha_{s_1}^{f_1}(T_{s_1}^{1.o.f_1}) = 5 \cdot 20 + 125 = 225$	
	=		= $\gamma_{5,225}$

arrivalBound($s_5, \{f_1\}, \{f_0\}) = \alpha_{s_5}^{f_1}$		FIFO_MUX	ARB_MUX
$\alpha_{s_2}^{f_1}$			= $\gamma_{5,225}$
$\alpha_{s_2}^{x(f_1)}$			= $\gamma_{0,0}$
$\beta_{s_2}^{1.o.f_1} = \beta_{s_2} \ominus \alpha_{s_2}^{x(f_1)} = \beta_{R_{s_2}^{1.o.f_1}, T_{s_2}^{1.o.f_1}}$	=		= $\beta_{20,20}$
$\alpha_{s_5}^{f_1} = \alpha_{s_2}^{f_1} \circ \beta_{s_2}^{1.o.f_1} = \gamma_{r_{s_5}^{f_1}, b_{s_5}^{f_1}}$	$r_{s_5}^{f_1}$		= 5
	$b_{s_5}^{f_1}$	$\alpha_{s_2}^{f_1}(T_{s_2}^{1.o.f_1}) = 5 \cdot 20 + 225 = 325$	
	=		= $\gamma_{5,325}$

arrivalBound($s_4, \{f_2\}, \mathcal{G}$), $\mathcal{G} \in \mathcal{P}(\mathcal{F}) = \alpha_{s_4}^{f_2}$		FIFO_MUX	ARB_MUX
$\alpha_{s_3}^{f_2}$			= $\gamma_{5,25}$
$\alpha_{s_3}^{x(f_2)}$			= $\gamma_{0,0}$
$\beta_{s_3}^{1.o.f_2} = \beta_{s_3} \ominus \alpha_{s_3}^{x(f_2)} = \beta_{R_{s_3}^{1.o.f_2}, T_{s_3}^{1.o.f_2}}$	=		= $\beta_{20,20}$
$\alpha_{s_4}^{f_2} = \alpha_{s_3}^{f_2} \circ \beta_{s_3}^{1.o.f_2} = \gamma_{r_{s_4}^{f_2}, b_{s_4}^{f_2}}$	$r_{s_4}^{f_2}$		= 5
	$b_{s_4}^{f_2}$	$\alpha_{s_3}^{f_2}(T_{s_3}^{1.o.f_2}) = 5 \cdot 20 + 25 = 125$	
	=		= $\gamma_{5,125}$

arrivalBound($s_5, \{f_2\}, \mathcal{G}$), $\mathcal{G} \in \mathcal{P}(\mathcal{F}) = \alpha_{s_5}^{f_2}$		FIFO_MUX	ARB_MUX
$\alpha_{s_4}^{f_2}$			= $\gamma_{5,125}$
$\alpha_{s_4}^{x(f_2)}$			= $\gamma_{0,0}$
$\beta_{s_4}^{1.o.f_2} = \beta_{s_4} \ominus \alpha_{s_4}^{x(f_2)} = \beta_{R_{s_4}^{1.o.f_2}, T_{s_4}^{1.o.f_2}}$	=		= $\beta_{20,20}$
$\alpha_{s_5}^{f_2} = \alpha_{s_4}^{f_2} \circ \beta_{s_4}^{1.o.f_2} = \gamma_{r_{s_5}^{f_2}, b_{s_5}^{f_2}}$	$r_{s_5}^{f_2}$		= 5
	$b_{s_5}^{f_2}$	$\alpha_{s_4}^{f_2}(T_{s_4}^{1.o.f_2}) = 5 \cdot 20 + 125 = 225$	
	=		= $\gamma_{5,225}$

arrivalBound($s_6, \{f_1, f_2\}, \{f_0\}) = \alpha_{s_6}^{\{f_1, f_2\}}$		FIFO_MUX	ARB_MUX
$\alpha_{s_5}^{\{f_1, f_2\}}$			$= \gamma_{10,550}$
$\alpha_{s_5}^{x\{f_1, f_2\}}$			$= \gamma_{0,0}$
$\beta_{s_5}^{\text{l.o.}\{f_1, f_2\}} = \beta_{s_5} \ominus \alpha_{s_5}^{x\{f_1, f_2\}} = \beta_{R_{s_5}^{\text{l.o.}\{f_1, f_2\}}, T_{s_5}^{\text{l.o.}\{f_1, f_2\}}}$	$=$		$= \beta_{20,20}$
$\alpha_{s_6}^{\{f_1, f_2\}} = \alpha_{s_5}^{\{f_1, f_2\}} \oslash \beta_{s_5}^{\text{l.o.}\{f_1, f_2\}} = \gamma_{r_{s_6}^{\{f_1, f_2\}}, b_{s_6}^{\{f_1, f_2\}}}$	$r_{s_6}^{\{f_1, f_2\}}$		$= 10$
	$b_{s_6}^{\{f_1, f_2\}}$	$\alpha_{s_5}^{\{f_1, f_2\}} (T_{s_5}^{\text{l.o.}\{f_1, f_2\}})$	$= 10 \cdot 20 + 550 = 750$
	$=$		$= \gamma_{10,750}$

arrivalBound($s_2, \{f_0, f_1\}, \mathcal{G}), \mathcal{G} \in \mathcal{P}(\{f_2\}) = \alpha_{s_2}^{\{f_0, f_1\}}$		FIFO_MUX	ARB_MUX
$\alpha_{s_1}^{\{f_0, f_1\}}$			$= \gamma_{10,150}$
$\alpha_{s_1}^{x\{f_0, f_1\}}$			$= \gamma_{0,0}$
$\beta_{s_1}^{\text{l.o.}\{f_0, f_1\}} = \beta_{s_1} \ominus \alpha_{s_1}^{x\{f_0, f_1\}} = \beta_{R_{s_1}^{\text{l.o.}\{f_0, f_1\}}, T_{s_1}^{\text{l.o.}\{f_0, f_1\}}}$	$=$		$= \beta_{20,20}$
$\alpha_{s_2}^{\{f_0, f_1\}} = \alpha_{s_1}^{\{f_0, f_1\}} \oslash \beta_{s_1}^{\text{l.o.}\{f_0, f_1\}} = \gamma_{r_{s_2}^{\{f_0, f_1\}}, b_{s_2}^{\{f_0, f_1\}}}$	$r_{s_2}^{\{f_0, f_1\}}$		$= 10$
	$b_{s_2}^{\{f_0, f_1\}}$	$\alpha_{s_1}^{\{f_0, f_1\}} (T_{s_1}^{\text{l.o.}\{f_0, f_1\}})$	$= 10 \cdot 20 + 150 = 350$
	$=$		$= \gamma_{10,350}$

arrivalBound($s_5, \{f_0, f_1\}, \mathcal{G}), \mathcal{G} \in \mathcal{P}(\{f_2\}) = \alpha_{s_5}^{\{f_0, f_1\}}$		FIFO_MUX	ARB_MUX
$\alpha_{s_2}^{\{f_0, f_1\}}$			$= \gamma_{10,350}$
$\alpha_{s_2}^{x\{f_0, f_1\}}$			$= \gamma_{0,0}$
$\beta_{s_2}^{\text{l.o.}\{f_0, f_1\}} = \beta_{s_2} \ominus \alpha_{s_2}^{x\{f_0, f_1\}} = \beta_{R_{s_2}^{\text{l.o.}\{f_0, f_1\}}, T_{s_2}^{\text{l.o.}\{f_0, f_1\}}}$	$=$		$= \beta_{20,20}$
$\alpha_{s_5}^{\{f_0, f_1\}} = \alpha_{s_2}^{\{f_0, f_1\}} \oslash \beta_{s_2}^{\text{l.o.}\{f_0, f_1\}} = \gamma_{r_{s_5}^{\{f_0, f_1\}}, b_{s_5}^{\{f_0, f_1\}}}$	$r_{s_5}^{\{f_0, f_1\}}$		$= 10$
	$b_{s_5}^{\{f_0, f_1\}}$	$\alpha_{s_2}^{\{f_0, f_1\}} (T_{s_2}^{\text{l.o.}\{f_0, f_1\}})$	$= 10 \cdot 20 + 350 = 550$
	$=$		$= \gamma_{10,550}$

arrivalBound($s_6, \{f_0, f_1, f_2\}, \{\}) = \alpha_{s_6}^{\{f_0, f_1, f_2\}}$		FIFO_MUX	ARB_MUX
$\alpha_{s_5}^{\{f_0, f_1, f_2\}}$			$= \gamma_{10,775}$
$\alpha_{s_5}^{x\{f_0, f_1, f_2\}}$			$= \gamma_{0,0}$
$\beta_{s_5}^{\text{l.o.}\{f_0, f_1, f_2\}} = \beta_{s_5} \ominus \alpha_{s_5}^{x\{f_0, f_1, f_2\}} = \beta_{R_{s_5}^{\text{l.o.}\{f_0, f_1, f_2\}}, T_{s_5}^{\text{l.o.}\{f_0, f_1, f_2\}}}$	$=$		$= \beta_{20,20}$
$\alpha_{s_6}^{\{f_0, f_1, f_2\}} = \alpha_{s_5}^{\{f_0, f_1, f_2\}} \oslash \beta_{s_5}^{\text{l.o.}\{f_0, f_1, f_2\}} = \gamma_{r_{s_6}^{\{f_0, f_1, f_2\}}, b_{s_6}^{\{f_0, f_1, f_2\}}}$	$r_{s_6}^{\{f_0, f_1, f_2\}}$		$= 15$
	$b_{s_6}^{\{f_0, f_1, f_2\}}$	$\alpha_{s_5}^{\{f_0, f_1, f_2\}} (T_{s_5}^{\text{l.o.}\{f_0, f_1, f_2\}})$	$= 15 \cdot 20 + 775 = 1075$
	$=$		$= \gamma_{15,1075}$

Flow f_0

TFA		FIFO_MUX	ARB_MUX
s_1	$\alpha_{s_1} = \alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_1}$	$= \gamma_{5,25} + \gamma_{5,125} = \gamma_{10,150}$	
	$D_{s_1}^{f_0}$	$\beta_{s_1} = b_{s_1}$ $20 \cdot [t - 20]^+ = 150$ $t = 27\frac{1}{2}$	$\beta_{s_1} = \alpha_{s_1}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 150$ $t = 55$
	$B_{s_1}^{f_0}$	$\alpha_{s_1}(T_{s_1}) = 10 \cdot 20 + 150 = 350$	
s_2	$\alpha_{s_2} = \alpha_{s_2}^{\{f_0, f_1\}}$	$= \gamma_{10,350}$	
	$D_{s_2}^{f_0}$	$\beta_{s_2} = b_{s_2}$ $20 \cdot [t - 20]^+ = 350$ $t = 37\frac{1}{2}$	$\beta_{s_2} = \alpha_{s_2}$ $20 \cdot [t - 20]^+ = 10 \cdot t + 350$ $t = 75$
	$B_{s_2}^{f_0}$	$\alpha_{s_2}(T_{s_2}) = 10 \cdot 20 + 350 = 550$	
s_5	$\alpha_{s_5} = \alpha_{s_5}^{\{f_0, f_1\}} + \alpha_{s_5}^{f_2}$	$= \gamma_{10,550} + \gamma_{5,225} = \gamma_{15,775}$	
	$D_{s_5}^{f_0}$	$\beta_{s_5} = b_{s_5}$ $20 \cdot [t - 20]^+ = 775$ $t = 58\frac{3}{4}$	$\beta_{s_5} = \alpha_{s_5}$ $20 \cdot [t - 20]^+ = 15 \cdot t + 775$ $t = 235$
	$B_{s_5}^{f_0}$	$\alpha_{s_5}(T_{s_5}) = 15 \cdot 20 + 775 = 1075$	
s_6	$\alpha_{s_6} = \alpha_{s_6}^{\{f_0, f_1, f_2\}}$	$= \gamma_{15,1075}$	
	$D_{s_6}^{f_0}$	$\beta_{s_6} = b_{s_6}$ $20 \cdot [t - 20]^+ = 1075$ $t = 73\frac{3}{4}$	$\beta_{s_6} = \alpha_{s_6}$ $20 \cdot [t - 20]^+ = 15 \cdot t + 1075$ $t = 295$
	$B_{s_6}^{f_0}$	$\alpha_{s_6}(T_{s_6}) = 15 \cdot 20 + 1075 = 1375$	
D^{f_0}	$\sum_{i=\{1,2,5,6\}} D_{s_i}^{f_0} = 27\frac{1}{2} + 37\frac{1}{2} + 58\frac{3}{4} + 73\frac{3}{4} = 197\frac{1}{2}$		
B^{f_0}	$\max_{i=\{1,2,5,6\}} B_{s_i}^{f_0} = 1375$		

$$\sum_{i=\{1,2,5,6\}} D_{s_i}^{f_0} = 55 + 75 + 235 + 295 = 660$$

SFA FIFO_MUX:

$$\begin{aligned}
\beta_{e2e}^{l.o.f_0} &= \left(\beta_{s_1}^{l.o.x(f_0)} \ominus \alpha_{s_1}^{x(f_0)} \right) \otimes \left(\beta_{s_2}^{l.o.x(f_0)} \ominus \alpha_{s_2}^{x(f_0)} \right) \otimes \left(\beta_{s_5}^{l.o.x(f_0)} \ominus \alpha_{s_5}^{x(f_0)} \right) \otimes \left(\beta_{s_6}^{l.o.x(f_0)} \ominus \alpha_{s_6}^{x(f_0)} \right) \\
&= \left(\beta_{s_1}^{l.o.x(f_0)} \ominus \alpha_{s_1}^{x(f_0)} \right) \otimes \left(\beta_{s_2}^{l.o.x(f_0)} \ominus \alpha_{s_2}^{x(f_0)} \right) \otimes \left(\beta_{s_5}^{l.o.x(f_0)} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \right) \otimes \left(\beta_{s_6}^{l.o.x(f_0)} \ominus \alpha_{s_6}^{\{f_1, f_2\}} \right) \\
&= \left(\beta_{s_1} \ominus \alpha_{s_1}^{f_1} \right) \otimes \left(\beta_{s_2} \ominus \alpha_{s_2}^{f_1} \right) \otimes \left(\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \right) \otimes \left(\beta_{s_6} \ominus \left(\alpha_{s_5}^{\{f_1, f_2\}} \circ \beta_{s_5}^{l.o.\{f_1, f_2\}} \right) \right) \\
&= \left(\beta_{s_1} \ominus \alpha_{s_1}^{f_1} \right) \otimes \left(\beta_{s_2} \ominus \alpha_{s_2}^{f_1} \right) \otimes \left(\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \right) \otimes \left(\beta_{s_6} \ominus \left((\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \circ \beta_{s_5}^{l.o.\{f_1, f_2\}} \right) \right) \\
&= \left(\beta_{s_1} \ominus \alpha_{s_1}^{f_1} \right) \otimes \left(\beta_{s_2} \ominus \alpha_{s_2}^{f_1} \right) \otimes \left(\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \right) \otimes \left(\beta_{s_6} \ominus \left((\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \circ \left(\beta_{s_5} \ominus \alpha_{s_5}^{x\{f_1, f_2\}} \right) \right) \right) \\
&= \left(\beta_{s_1} \ominus \alpha_{s_1}^{f_1} \right) \otimes \left(\beta_{s_2} \ominus \alpha_{s_2}^{f_1} \right) \otimes \left(\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \right) \otimes \left(\beta_{s_6} \ominus \left((\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \circ \beta_{s_5} \right) \right) \\
&= (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus \gamma_{5,225}) \otimes (\beta_{20,20} \ominus (\gamma_{5,325} + \gamma_{5,225})) \otimes (\beta_{20,20} \ominus ((\gamma_{5,325} + \gamma_{5,225}) \circ \beta_{20,20})) \\
&= (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus \gamma_{5,225}) \otimes (\beta_{20,20} \ominus \gamma_{10,550}) \otimes (\beta_{20,20} \ominus (\gamma_{10,550} \circ \beta_{20,20})) \\
&= (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus \gamma_{5,225}) \otimes (\beta_{20,20} \ominus \gamma_{10,550}) \otimes (\beta_{20,20} \ominus \gamma_{10,750}) \\
&= \beta_{15,26\frac{1}{4}} \otimes \beta_{15,31\frac{1}{4}} \otimes \beta_{10,47\frac{1}{2}} \otimes \beta_{10,57\frac{1}{2}} \\
&= \beta_{10,162\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
D^{f_0} &= \frac{R_{e2e}^{l.o.f_0} \cdot T_{e2e}^{l.o.f_0} + b^{f_0}}{R_{e2e}^{l.o.f_0}} \\
&= \frac{10 \cdot 162\frac{1}{2} + 25}{10} \\
&= 165
\end{aligned}$$

$$\begin{aligned}
B^{f_0} &= \alpha^{f_0} (T_{e2e}^{l.o.f_0}) \\
&= 5 \cdot 162\frac{1}{2} + 25 \\
&= 837\frac{1}{2}
\end{aligned}$$

SFA ARB_MUX:

$$\begin{aligned}
\beta_{e2e}^{l.o.f_0} &= \left(\beta_{s_1}^{l.o.x(f_0)} \ominus \alpha_{s_1}^{x(f_0)} \right) \otimes \left(\beta_{s_2}^{l.o.x(f_0)} \ominus \alpha_{s_2}^{x(f_0)} \right) \otimes \left(\beta_{s_5}^{l.o.x(f_0)} \ominus \alpha_{s_5}^{x(f_0)} \right) \otimes \left(\beta_{s_6}^{l.o.x(f_0)} \ominus \alpha_{s_6}^{x(f_0)} \right) \\
&= \left(\beta_{s_1}^{l.o.x(f_0)} \ominus \alpha_{s_1}^{x(f_0)} \right) \otimes \left(\beta_{s_2}^{l.o.x(f_0)} \ominus \alpha_{s_2}^{x(f_0)} \right) \otimes \left(\beta_{s_5}^{l.o.x(f_0)} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \right) \otimes \left(\beta_{s_6}^{l.o.x(f_0)} \ominus \alpha_{s_6}^{\{f_1, f_2\}} \right) \\
&= \left(\beta_{s_1} \ominus \alpha_{s_1}^{f_1} \right) \otimes \left(\beta_{s_2} \ominus \alpha_{s_2}^{f_1} \right) \otimes \left(\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \right) \otimes \left(\beta_{s_6} \ominus \left(\alpha_{s_5}^{\{f_1, f_2\}} \oslash \beta_{s_5}^{l.o.\{f_1, f_2\}} \right) \right) \\
&= \left(\beta_{s_1} \ominus \alpha_{s_1}^{f_1} \right) \otimes \left(\beta_{s_2} \ominus \alpha_{s_2}^{f_1} \right) \otimes \left(\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \right) \otimes \left(\beta_{s_6} \ominus \left((\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \oslash \beta_{s_5}^{l.o.\{f_1, f_2\}} \right) \right) \\
&= \left(\beta_{s_1} \ominus \alpha_{s_1}^{f_1} \right) \otimes \left(\beta_{s_2} \ominus \alpha_{s_2}^{f_1} \right) \otimes \left(\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \right) \otimes \left(\beta_{s_6} \ominus \left((\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \oslash \left(\beta_{s_5} \ominus \alpha_{s_5}^{x\{f_1, f_2\}} \right) \right) \right) \\
&= \left(\beta_{s_1} \ominus \alpha_{s_1}^{f_1} \right) \otimes \left(\beta_{s_2} \ominus \alpha_{s_2}^{f_1} \right) \otimes \left(\beta_{s_5} \ominus (\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \right) \otimes \left(\beta_{s_6} \ominus \left((\alpha_{s_5}^{f_1} + \alpha_{s_5}^{f_2}) \oslash \beta_{s_5} \right) \right) \\
&= (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus \gamma_{5,225}) \otimes (\beta_{20,20} \ominus (\gamma_{5,325} + \gamma_{5,225})) \otimes (\beta_{20,20} \ominus ((\gamma_{5,325} + \gamma_{5,225}) \oslash \beta_{20,20})) \\
&= (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus \gamma_{5,225}) \otimes (\beta_{20,20} \ominus \gamma_{10,550}) \otimes (\beta_{20,20} \ominus (\gamma_{10,550} \oslash \beta_{20,20})) \\
&= (\beta_{20,20} \ominus \gamma_{5,125}) \otimes (\beta_{20,20} \ominus \gamma_{5,225}) \otimes (\beta_{20,20} \ominus \gamma_{10,550}) \otimes (\beta_{20,20} \ominus \gamma_{10,750}) \\
&= \beta_{15,35} \otimes \beta_{15,41\frac{2}{3}} \otimes \beta_{10,95} \otimes \beta_{10,115} \\
&= \beta_{10,286\frac{2}{3}}
\end{aligned}$$

$$\begin{aligned}
D^{f_0} &= \frac{R_{e2e}^{l.o.f_0} \cdot T_{e2e}^{l.o.f_0} + b^{f_0}}{R_{e2e}^{l.o.f_0}} \\
&= \frac{10 \cdot 286\frac{2}{3} + 25}{10} \\
&= 289\frac{1}{6}
\end{aligned}$$

$$\begin{aligned}
B^{f_0} &= \alpha^{f_0} (T_{e2e}^{l.o.f_0}) \\
&= 5 \cdot 286\frac{2}{3} + 25 \\
&= 1458\frac{1}{3}
\end{aligned}$$

PMOO		ARB_MUX
s ₁	$\frac{\bar{x}(f_0)}{\alpha_{s_1}}$	= $\gamma_{5,125}$
	$\frac{x(f_0)}{\alpha_{s_1}}$	= $\gamma_{5,125}$
s ₂	$\frac{\bar{x}(f_0)}{\alpha_{s_2}}$	= $\gamma_{0,0}$
	$\frac{x(f_0)}{\alpha_{s_2}}$	= $\gamma_{5,125}$
s ₅	$\frac{\bar{x}(f_0)}{\alpha_{s_5}}$	= $\gamma_{5,225}$
	$\frac{x(f_0)}{\alpha_{s_5}}$	= $\gamma_{10,xxx}$
s ₆	$\frac{\bar{x}(f_0)}{\alpha_{s_6}}$	= $\gamma_{0,0}$
	$\frac{x(f_0)}{\alpha_{s_6}}$	= $\gamma_{10,xxx}$
$\beta_{e2e}^{1.o.f_0} = \beta_{R_{e2e}^{1.o.f_0}, T_{e2e}^{1.o.f_0}}$	$R_{e2e}^{1.o.f_0} = \bigwedge_{i \in \{1,2,5,6\}} (R_{s_i} - r_{s_i}^{x(f_0)})$	= $(20 - 5) \wedge (20 - 5) \wedge (20 - 10) \wedge (20 - 10)$ = 10
	$T_{e2e}^{1.o.f_0} = \sum_{i \in \{1,2,5,6\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_0)} + r_{s_i}^{x(f_0)} \cdot T_{s_i}}{R_{e2e}^{1.o.f_0}} \right)$	= $20 + \frac{125 + 5 \cdot 20}{10} + 20 + \frac{0 + 5 \cdot 20}{10} + 20 + \frac{225 + 10 \cdot 20}{10} + 20 + \frac{0 + 10 \cdot 20}{10}$ = $80 + \frac{950}{10}$ = 175
	=	= $\beta_{10,185}$
D^{f_0}	$\beta_{e2e}^{1.o.f_0} = b^{f_0}$ $10 \cdot [t - 175]^+ = 25$ $t = 177 \frac{1}{2}$	
B^{f_0}	$\alpha^{f_0}(T_{e2e}^{1.o.f_0}) = 5 \cdot 175 + 25$ = 900	

Flow f_1

TFA		FIFO_MUX	ARB_MUX
s_0	$\alpha_{s_1} = \alpha_{s_1}^{f_1}$		$= \gamma_{5,25}$
	$D_{s_0}^{f_1}$	$\beta_{s_0} = b_{s_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	FIFO per micro flow $\beta_{s_0} = b_{s_0}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$
	$B_{s_0}^{f_1}$		$\alpha_{s_0}(T_{s_0}) = 5 \cdot 20 + 25 = 125$
s_1	$\alpha_{s_1} = \alpha_{s_1}^{f_0} + \alpha_{s_1}^{f_1}$		$= \gamma_{5,25} + \gamma_{5,125} = \gamma_{10,150}$
	$D_{s_1}^{f_1}$	$= 27\frac{1}{2}$	$= 55$
	$B_{s_1}^{f_1}$		$= 350$
s_2	$\alpha_{s_2} = \alpha_{s_2}^{\{f_0, f_1\}}$		$= \gamma_{10,350}$
	$D_{s_2}^{f_1}$	$= 37\frac{1}{2}$	$= 75$
	$B_{s_2}^{f_1}$		$= 550$
s_5	$\alpha_{s_5} = \alpha_{s_5}^{\{f_0, f_1\}} + \alpha_{s_5}^{f_2}$		$= \gamma_{5,225} + \gamma_{10,550} = \gamma_{15,775}$
	$D_{s_5}^{f_1}$	$= 58\frac{3}{4}$	$= 235$
	$B_{s_5}^{f_1}$		$= 1075$
s_6	$\alpha_{s_6} = \alpha_{s_6}^{\{f_0, f_1, f_2\}}$		$= \gamma_{15,1075}$
	$D_{s_6}^{f_1}$	$= 73\frac{3}{4}$	$= 295$
	$B_{s_6}^{f_1}$		$= 1375$
D^{f_1}	$\sum_{i \in \{0,1,2,5,6\}} D_{s_i}^{f_1} = 21\frac{1}{4} + 27\frac{1}{2} + 37\frac{1}{2} + 58\frac{3}{4} + 73\frac{3}{4} = 218\frac{3}{4}$		
B^{f_1}	$\sum_{i \in \{0,1,2,5,6\}} D_{s_i}^{f_1} = 21\frac{1}{4} + 55 + 75 + 235 + 295 = 681\frac{1}{4}$ $\max_{i \in \{0,1,2,5,6\}} B_{s_i}^{f_1} = 1375$		

PMOO		ARB_MUX
s_0	$\bar{x}(f_1)$ α_{s_0}	$= \gamma_{0,0}$
	$x(f_1)$ α_{s_0}	$= \gamma_{0,0}$
s_1	$\bar{x}(f_1)$ α_{s_1}	$= \gamma_{5,25}$
	$x(f_1)$ α_{s_1}	$= \gamma_{5,25}$
s_2	$\bar{x}(f_1)$ α_{s_2}	$= \gamma_{0,0}$
	$x(f_1)$ α_{s_2}	$= \gamma_{5,125}$
s_5	$\bar{x}(f_1)$ α_{s_5}	$= \gamma_{5,225}$
	$x(f_1)$ α_{s_5}	$= \gamma_{10,xxx}$
s_6	$\bar{x}(f_1)$ α_{s_6}	$= \gamma_{0,0}$
	$x(f_1)$ α_{s_6}	$= \gamma_{10,xxx}$
$\beta_{e2e}^{l.o.f_1} = \beta_{R_{e2e}^{l.o.f_1}, T_{e2e}^{l.o.f_1}}$	$R_{e2e}^{l.o.f_1} = \bigwedge_{i \in \{0,1,2,5,6\}} (R_{s_i} - r_{s_i}^{x(f_1)})$	$= (20 - 0) \wedge (20 - 5) \wedge (20 - 5) \wedge (20 - 10) \wedge (20 - 10)$ $= 10$
	$T_{e2e}^{l.o.f_1} = \sum_{i \in \{0,1,2,5,6\}} \left(T_{s_i} + \frac{b_{s_i}^{x(f_1)} + r_{s_i}^{x(f_1)} \cdot T_{s_i}}{R_{e2e}^{l.o.f_1}} \right)$	$= 20 + \frac{0 + 0 \cdot 20}{10} + 20 + \frac{25 + 5 \cdot 20}{10} + 20 + \frac{0 + 5 \cdot 20}{10} + 20 + \frac{225 + 10 \cdot 20}{10} + 20 + \frac{0 + 10 \cdot 20}{10}$ $= 100 + \frac{850}{10}$ $= 185$
	$=$	$= \beta_{10,185}$
Df_1		$\beta_{e2e}^{l.o.f_1} = b^{f_1}$ $10 \cdot [t - 185]^+ = 25$ $t = 187\frac{1}{2}$
Bf_1		$\alpha^{f_1}(T_{e2e}^{l.o.f_1}) = 5 \cdot 185 + 25$ $= 950$

Flow f_2

	TFA	FIFO_MUX	ARB_MUX
s_3	$\alpha_{s_3} = \alpha_{s_3}^{f_2}$	$= \gamma_{5,25}$	
	$Df_{s_3}^2$	$\beta_{s_3} = b_{s_3}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$	FIFO per micro flow $\beta_{s_3} = b_{s_3}$ $20 \cdot [t - 20]^+ = 25$ $t = 21\frac{1}{4}$
	$Bf_{s_3}^2$	$\alpha_{s_3}(T_{s_3}) = 5 \cdot 20 + 25 = 125$	
s_4	$\alpha_{s_4} = \alpha_{s_4}^{f_2}$	$= \gamma_{5,125}$	
	$Df_{s_4}^2$	$\beta_{s_4} = b_{s_4}$ $20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$	FIFO per micro flow $\beta_{s_4} = b_{s_4}$ $20 \cdot [t - 20]^+ = 125$ $t = 26\frac{1}{4}$
	$Bf_{s_4}^2$	$\alpha_{s_4}(T_{s_4}) = 5 \cdot 20 + 125 = 225$	
s_5	$\alpha_{s_5} = \alpha_{s_5}^{\{f_0, f_1\}} + \alpha_{s_5}^{f_2}$	$= \gamma_{5,225} + \gamma_{10,550} = \gamma_{15,775}$	
	$Df_{s_5}^2$	$= 58\frac{3}{4}$	$= 235$
	$Bf_{s_5}^2$	$= 1075$	
s_6	$\alpha_{s_6} = \alpha_{s_6}^{\{f_0, f_1, f_2\}}$	$= \gamma_{15,1075}$	
	$Df_{s_6}^2$	$= 73\frac{3}{4}$	$= 295$
	$Bf_{s_6}^2$	$= 1375$	
	Df^2	$\sum_{i \in \{3,4,5,6\}} Df_{s_i}^2 = 180$	$\sum_{i \in \{3,4,5,6\}} Df_{s_i}^2 = 577\frac{1}{2}$
	Bf^2	$\max_{i \in \{3,4,5,6\}} Bf_{s_i}^2 = 1375$	

SFA FIFO_MUX:

$$\begin{aligned}
\beta_{e2e}^{l.o.f2} &= \left(\beta_{s3}^{l.o.x(f2)} \ominus \alpha_{s3}^{x(f2)} \right) \otimes \left(\beta_{s4}^{l.o.x(f2)} \ominus \alpha_{s4}^{x(f2)} \right) \otimes \left(\beta_{s5}^{l.o.x(f2)} \ominus \alpha_{s5}^{x(f2)} \right) \otimes \left(\beta_{s6}^{l.o.x(f2)} \ominus \alpha_{s6}^{x(f2)} \right) \\
&= \beta_{s3} \otimes \beta_{s4} \otimes \left(\left(\beta_{s5} \ominus \alpha_{s5}^{x(f2)} \right) \ominus \alpha_{s5}^{\{f0,f1\}} \right) \otimes \left(\beta_{s6}^{l.o.x(f2)} \ominus \left(\alpha_{s5}^{x(f2)} \circ \beta_{s5}^{l.o.x(f2)} \right) \right) \\
&= \beta_{s3} \otimes \beta_{s4} \otimes \left(\beta_{s5} \ominus \alpha_{s5}^{\{f0,f1\}} \right) \otimes \left(\beta_{s6} \ominus \left(\alpha_{s5}^{\{f0,f1\}} \circ \beta_{s5} \right) \right) \\
&= \beta_{s3} \otimes \beta_{s4} \otimes \left(\beta_{s5} \ominus \left(\alpha_{s1}^{\{f0,f1\}} \circ (\beta_{s1} \otimes \beta_{s2}) \right) \right) \otimes \left(\beta_{s6} \ominus \left(\left(\alpha_{s1}^{\{f0,f1\}} \circ (\beta_{s1} \otimes \beta_{s2}) \right) \circ \beta_{s5} \right) \right) \\
&= \beta_{s3} \otimes \beta_{s4} \otimes \left(\beta_{s5} \ominus \left((\alpha^{f1} \circ \beta_{s0}) + \alpha^{f0} \right) \circ (\beta_{s1} \otimes \beta_{s2}) \right) \otimes \left(\beta_{s6} \ominus \left(\left((\alpha^{f1} \circ \beta_{s0}) + \alpha^{f0} \right) \circ (\beta_{s1} \otimes \beta_{s2}) \right) \circ \beta_{s5} \right) \\
&= \beta_{20,20} \otimes \beta_{20,20} \otimes \left(\beta_{20,20} \ominus \left((\gamma_{5,25} \circ \beta_{20,20}) + \gamma_{5,25} \right) \circ (\beta_{20,20} \otimes \beta_{20,20}) \right) \otimes \left(\beta_{20,20} \ominus \left(\left((\gamma_{5,25} \circ \beta_{20,20}) + \gamma_{5,25} \right) \circ (\beta_{20,20} \otimes \beta_{20,20}) \right) \circ \beta_{20,20} \right) \\
&= \beta_{20,40} \otimes \left(\beta_{20,20} \ominus \left((\gamma_{5,25} \circ \beta_{20,20}) + \gamma_{5,25} \right) \circ \beta_{20,40} \right) \otimes \left(\beta_{20,20} \ominus \left(\left((\gamma_{5,25} \circ \beta_{20,20}) + \gamma_{5,25} \right) \circ \beta_{20,40} \right) \circ \beta_{20,20} \right) \\
&= \beta_{20,40} \otimes \left(\beta_{20,20} \ominus \left((\gamma_{5,125} + \gamma_{5,25}) \circ \beta_{20,40} \right) \right) \otimes \left(\beta_{20,20} \ominus \left((\gamma_{5,125} + \gamma_{5,25}) \circ \beta_{20,40} \right) \circ \beta_{20,20} \right) \\
&= \beta_{20,40} \otimes \left(\beta_{20,20} \ominus (\gamma_{10,150} \circ \beta_{20,40}) \right) \otimes \left(\beta_{20,20} \ominus \left((\gamma_{10,150} \circ \beta_{20,40}) \circ \beta_{20,20} \right) \right) \\
&= \beta_{20,40} \otimes \left(\beta_{20,20} \ominus \gamma_{10,550} \right) \otimes \left(\beta_{20,20} \ominus (\gamma_{10,550} \circ \beta_{20,20}) \right) \\
&= \beta_{20,40} \otimes \left(\beta_{20,20} \ominus \gamma_{10,550} \right) \otimes \left(\beta_{20,20} \ominus \gamma_{10,750} \right) \\
&= \beta_{20,40} \otimes \beta_{10,47\frac{1}{2}} \otimes \beta_{10,55\frac{1}{2}} \\
&= \beta_{10,145}
\end{aligned}$$

$$\begin{aligned}
D^{f1} &= \frac{R_{e2e}^{l.o.f1} \cdot T_{e2e}^{l.o.f1} + b^{f1}}{R_{e2e}^{l.o.f1}} \\
&= \frac{10 \cdot 145 + 25}{10} \\
&= 147\frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
B^{f1} &= \alpha^{f1} (T_{e2e}^{l.o.f1}) \\
&= 5 \cdot 145 + 25 \\
&= 750
\end{aligned}$$

SFA ARB_MUX:

$$\begin{aligned}
\beta_{e2e}^{l.o.f2} &= \left(\beta_{s3}^{l.o.x(f2)} \ominus \alpha_{s3}^{x(f2)} \right) \otimes \left(\beta_{s4}^{l.o.x(f2)} \ominus \alpha_{s4}^{x(f2)} \right) \otimes \left(\beta_{s5}^{l.o.x(f2)} \ominus \alpha_{s5}^{x(f2)} \right) \otimes \left(\beta_{s6}^{l.o.x(f2)} \ominus \alpha_{s6}^{x(f2)} \right) \\
&= \beta_{s3} \otimes \beta_{s4} \otimes \left(\left(\beta_{s5} \ominus \alpha_{s5}^{x(f2)} \right) \ominus \alpha_{s5}^{\{f0,f1\}} \right) \otimes \left(\beta_{s6}^{l.o.x(f2)} \ominus \left(\alpha_{s5}^{x(f2)} \oslash \beta_{s5}^{l.o.x(f2)} \right) \right) \\
&= \beta_{s3} \otimes \beta_{s4} \otimes \left(\beta_{s5} \ominus \alpha_{s5}^{\{f0,f1\}} \right) \otimes \left(\beta_{s6} \ominus \left(\alpha_{s5}^{\{f0,f1\}} \oslash \beta_{s5} \right) \right) \\
&= \beta_{s3} \otimes \beta_{s4} \otimes \left(\beta_{s5} \ominus \left(\alpha_{s1}^{\{f0,f1\}} \oslash (\beta_{s1} \otimes \beta_{s2}) \right) \right) \otimes \left(\beta_{s6} \ominus \left(\left(\alpha_{s1}^{\{f0,f1\}} \oslash (\beta_{s1} \otimes \beta_{s2}) \right) \oslash \beta_{s5} \right) \right) \\
&= \beta_{s3} \otimes \beta_{s4} \otimes \left(\beta_{s5} \ominus \left((\alpha^{f1} \oslash \beta_{s0}) + \alpha^{f0} \right) \oslash (\beta_{s1} \otimes \beta_{s2}) \right) \otimes \left(\beta_{s6} \ominus \left(\left((\alpha^{f1} \oslash \beta_{s0}) + \alpha^{f0} \right) \oslash (\beta_{s1} \otimes \beta_{s2}) \right) \oslash \beta_{s5} \right) \\
&= \beta_{20,20} \otimes \beta_{20,20} \otimes \left(\beta_{20,20} \ominus \left((\gamma_{5,25} \oslash \beta_{20,20}) + \gamma_{5,25} \right) \oslash (\beta_{20,20} \otimes \beta_{20,20}) \right) \otimes \left(\beta_{20,20} \ominus \left(\left((\gamma_{5,25} \oslash \beta_{20,20}) + \gamma_{5,25} \right) \oslash (\beta_{20,20} \otimes \beta_{20,20}) \right) \oslash \beta_{20,20} \right) \\
&= \beta_{20,40} \otimes \left(\beta_{20,20} \ominus \left((\gamma_{5,25} \oslash \beta_{20,20}) + \gamma_{5,25} \right) \oslash \beta_{20,40} \right) \otimes \left(\beta_{20,20} \ominus \left(\left((\gamma_{5,25} \oslash \beta_{20,20}) + \gamma_{5,25} \right) \oslash \beta_{20,40} \right) \oslash \beta_{20,20} \right) \\
&= \beta_{20,40} \otimes \left(\beta_{20,20} \ominus \left((\gamma_{5,125} + \gamma_{5,25}) \oslash \beta_{20,40} \right) \right) \otimes \left[\beta_{20,20} - \left((\gamma_{5,125} + \gamma_{5,25}) \oslash \beta_{20,40} \right) \oslash \beta_{20,20} \right]^+ \\
&= \beta_{20,40} \otimes \left(\beta_{20,20} \ominus (\gamma_{10,150} \oslash \beta_{20,40}) \right) \otimes \left(\beta_{20,20} \ominus \left((\gamma_{10,150} \oslash \beta_{20,40}) \oslash \beta_{20,20} \right) \right) \\
&= \beta_{20,40} \otimes \left(\beta_{20,20} \ominus \gamma_{10,550} \right) \otimes \left(\beta_{20,20} \ominus (\gamma_{10,550} \oslash \beta_{20,20}) \right) \\
&= \beta_{20,40} \otimes \left(\beta_{20,20} \ominus \gamma_{10,550} \right) \otimes \left(\beta_{20,20} \ominus \gamma_{10,750} \right) \\
&= \beta_{20,40} \otimes \beta_{10,95} \otimes \beta_{10,115} \\
&= \beta_{10,250}
\end{aligned}$$

$$\begin{aligned}
D^{f1} &= \frac{R_{e2e}^{l.o.f1} \cdot T_{e2e}^{l.o.f1} + b^{f1}}{R_{e2e}^{l.o.f1}} \\
&= \frac{10 \cdot 250 + 25}{10} \\
&= 252 \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
B^{f1} &= \alpha^{f1} (T_{e2e}^{l.o.f1}) \\
&= 5 \cdot 250 + 25 \\
&= 1275
\end{aligned}$$

PMOO		ARB_MUX
s ₃	$\bar{x}(f_2)$	
	α_{s_3}	= $\gamma_{0,0}$
s ₄	$x(f_2)$	
	α_{s_4}	= $\gamma_{0,0}$
s ₅	$\bar{x}(f_2)$	
	α_{s_5}	= $\gamma_{10,550}$
s ₆	$x(f_2)$	
	α_{s_6}	= $\gamma_{10,550}$
	$\bar{x}(f_2)$	
	α_{s_6}	= $\gamma_{0,0}$
	$x(f_2)$	
	α_{s_6}	= $\gamma_{10,xxx}$
$\beta_{e2e}^{1.o.f_2} = \beta_{R_{e2e}^{1.o.f_2}, T_{e2e}^{1.o.f_2}}$	$R_{e2e}^{1.o.f_2} = \bigwedge_{i \in \{3,4,5,6\}} (R_{s_i} - r_{s_i}^{x(f_2)})$	= $(20 - 0) \wedge (20 - 0) \wedge (20 - 10) \wedge (20 - 10)$
	$T_{e2e}^{1.o.f_2} = \sum_{i \in \{3,4,5,6\}} \left(T_{s_i} + \frac{b_{s_i}^{\bar{x}(f_2)} + r_{s_i}^{x(f_2)} \cdot T_{s_i}}{R_{e2e}^{1.o.f_2}} \right)$	= $20 + \frac{0 + 0 \cdot 20}{5} + 20 + \frac{0 + 0 \cdot 20}{5} + 20 + \frac{550 + 10 \cdot 20}{5} + 20 + \frac{0 + 10 \cdot 20}{5}$
		= $80 + \frac{950}{10}$
		= 175
	=	= $\beta_{10,175}$
D^{f_2}		$\beta_{e2e}^{1.o.f_2} = b^{f_2}$
		$10 \cdot [t - 175]^+ = 25$
		$t = 177 \frac{1}{2}$
B^{f_2}		$\alpha^{f_2}(T_{e2e}^{1.o.f_2}) = 5 \cdot 175 + 25$
		= 900