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**A Taxonomy for Multi-Period Resource Allocation  
Problems at System Edges  
(MPRASE Taxonomy)**

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## **Abstract**

Providing guaranteed QoS, be it statistical or deterministic, necessarily requires allocation of scarce resources. This might happen on a session or on an aggregate basis, nevertheless, it is conceivable that at least at system edges scarcity of resources, exposed in the form of non-negligible (virtual) costs, will prevail to necessitate explicit allocation of resources as opposed to pure overdimensioning. An example of this logic is constituted by the Differentiated Services (DiffServ) architecture which is largely based on explicit bilateral Service Level Agreements (SLA) between peering providers. Often such resource allocation decisions are done on a multi-period basis because resource allocation decisions at a certain point in time may depend on earlier decisions and thus it can turn out sub-optimal to look at decisions in an isolated fashion. In earlier papers we discussed the general class of optimization problems that are applicable in these scenarios. We call the class MPRASE (Multi-Period Resource Allocation at System Edges). In this paper we present a taxonomy for all the MPRASE problems.

# 1 Introduction

In this paper we present a taxonomy for MPRASE problems. MPRASE stands for Multi-Period Resource Allocation at System Edges and describes a class of optimization problems that are applicable to a number of scenarios in networking, e.g.

- RSVP/IntServ [4] over DiffServ/Bandwidth-Broker [5], [6].
- Admission control
- Reservation in advance
- Renegotiation for VBR streams [ 7]
- Cost minimal token bucket reservations

MPRASE problems and solution strategies are discussed in [2], [1], [3], [8] and further work.

## 2 MPRASE - Generalization & Taxonomy

In this section, we introduce a general structural model which tries to capture all the different facets of multi-period resource allocation at system edges (MPRASE) problems. This model then allows us to derive a taxonomy along its components which establishes the relations between the different problem incarnations.

### 2.1 Generalized Problem Structure Model

Figure 1 shows the overall structural model of the general class of MPRASE problems. Obviously, there are customers which generate requests towards several providers. These two groups are separated by a system edge on which an intermediary instance is located. The intermediary tries to mediate between the two by selecting providers on the one hand and enforcing admission control of the customers on the other hand. Note that the logical separation of the intermediary instance from customer and provider does not necessarily imply that it may not belong to either customer or provider premises. The requests are originated by the customers which desire a certain amount of resources offered by the providers. Furthermore, requests incur certain costs at the providers which need to be accounted for by customers. Several requests are generated in the course of time, thereby, reacting upon the dynamics of customers' demand. Let us now look at the different components/submodels of the structural model for MPRASE.

#### 2.1.1 Customer

The customer model of the MPRASE model captures the number of customers, i.e., if a single or multiple customers are considered, and the flexibility of the demand, i.e., whether demand may be dissatisfied or be served with a degraded quality. In the case that more customers exist the demand is the the product of the number of (served) customers

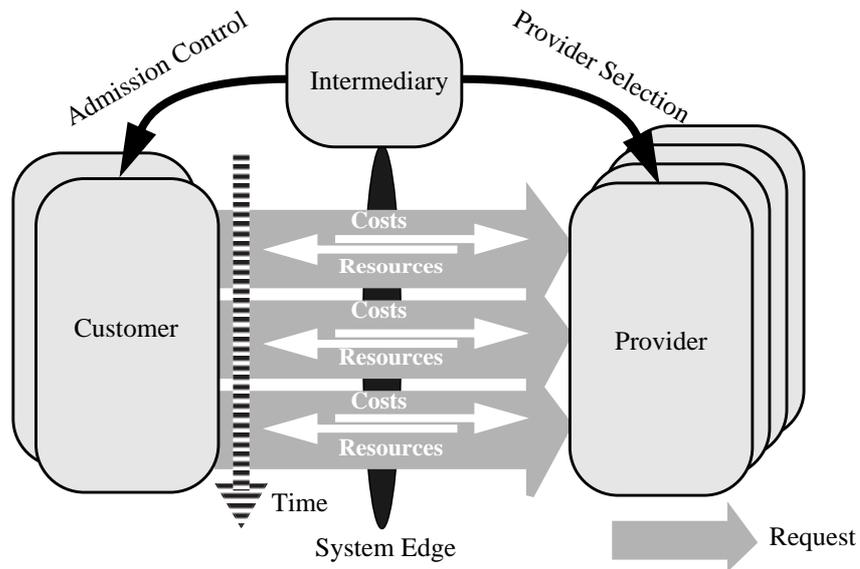


Figure 1: MPRASE problem structure.

and the amount of average demand of each individual customer. With the admission control mechanism the number of served customers is becoming variable while with degraded quality the amount of the demand that is satisfied by the provider is becoming flexible.

| Parameter                    | Value   | Abbreviation |
|------------------------------|---|--------------|
| <i>Number of Customer</i>    | <i>single Customer</i>                                      | <i>1</i>     |
|                              | <i>multiple Customers</i>                                   | <i>N</i>     |
| <i>Flexibility of Demand</i> | <i>inflexible (satisfied 100%)</i>                          | <i>-</i>     |
|                              | <i>dissatisfied/admission control (satisfied 0 or 100%)</i> | <i>AC</i>    |
|                              | <i>degraded quality (satisfied between 0 and 100%)</i>      | <i>DQ</i>    |

Table 1: Customer Model

The taxonomy for the customer model is displayed in Table 1. A single customer with inflexible demand would therefore be expressed by “1” while a model containing multiple customers that accept degraded quality are identified with “N<sub>DQ</sub>” or “N, DQ”.

### 2.1.2 Provider

The provider model encompasses the number of providers and whether they are modelled as having limited or unlimited capacity. While the latter is unrealistic it can be a simplifying, yet valid assumption for the case where supply exceeds demand with very high probability.

| Parameter                 | Value                     | Abbreviation |
|---------------------------|---------------------------|--------------|
| <i>Number of Provider</i> | <i>single Provider</i>    | <i>1</i>     |
|                           | <i>multiple Providers</i> | <i>N</i>     |
| <i>Capacity</i>           | <i>unlimited</i>          | -            |
|                           | <i>limited</i>            | <i>Cap</i>   |

Table 2: Provider Model

The taxonomy for the provider model is displayed in Table 1. A single provider with unlimited capacity would therefore be expressed by “1” while a model containing multiple customers with limited resources are identified with “ $N_{Cap}$ ”.

### 2.1.3 Resource

This component models the resources, i.e., whether they are one- or multidimensional or whether they are provided on a deterministic or statistical basis. Here, we make no particular assumption on the kind of guarantee with which resources are provided, i.e., whether they are statistically or deterministically available. Therefore, an allocation in our context does not necessarily mean an isolated, exclusive access to resources for a customer that made it.

| Parameter                   | Value                             | Abbreviation |
|-----------------------------|-----------------------------------|--------------|
| <i>Dimensions</i>           | <i>one-dimensional Resource</i>   | <i>1</i>     |
|                             | <i>multi-dimensional Resource</i> | $N_{Type}$   |
| <i>Stochastic Behaviour</i> | <i>deterministic</i>              | -            |
|                             | <i>statistical</i>                | <i>Stat</i>  |

Table 3: Resource Model

A one-dimensional deterministic resource like guaranteed bandwidth is expressed by “1”, a token bucket would be described by “ $N_{TB}$ ” or “ $N_{Token-Bucket}$ ”.

| Parameter            | Value               | Abbreviation |
|----------------------|---------------------|--------------|
| <i>Buffer + Rate</i> | <i>Token-Bucket</i> | <i>TB</i>    |

Table 4: Multi-Dimensional Resource Models

| Parameter | Value                           | Abbreviation |
|-----------|---------------------------------|--------------|
|           | <i>Multi-Level Token-Bucket</i> | <i>ML-TB</i> |
|           | <i>Leaky Bucket</i>             | <i>LB</i>    |
| ...       | ...                             |              |

Table 4: Multi-Dimensional Resource Models

### 2.1.4 Cost

The cost model seizes the cost structure for allocation requests, i.e., whether these incur certain fixed or transactional costs or whether the number of requests is just bounded and how variable costs for resource allocations are modelled, e.g., linearly or non-linearly. Please note that costs do not have to be monetary costs, they can also reflect imputed or fictive costs.

| Parameter  | Value                                   | Abbreviation |
|--|---|--------------|
| <i>Fixed Costs per reallocation</i>  | <i>Non-Existent</i>                     | -            |
|  | <i>Existent</i>                         | <i>F</i>     |
|  | <i>Infinite fixed Costs<sup>a</sup></i> | $F_{\infty}$ |
| <i>Variable Costs per amount of allocated resources per time</i>                   | <i>Non-Existent</i>                     | -            |
|  | <i>Existent</i>                         | <i>V</i>     |
| <i>Variable Costs per amount of used resources per time</i>                        | <i>Non-Existent</i>                     | -            |
|  | <i>Existent</i>                         | <i>U</i>     |
| <i>Variable Costs per amount of requested but not satisfied resources per time</i> | <i>Non-Existent</i>                     | -            |
|  | <i>Existent</i>                         | <i>R</i>     |
| <i>Variable Costs per accepted customer<sup>b</sup></i>                            | <i>Non-Existent</i>                     | -            |
|  | <i>Existent</i>                         | <i>C</i>     |

Table 5: Cost Model Elements

- a. Zero fixed costs for the first period and infinite fixed costs for all other periods. This effectively prohibits reallocations (and thus simplifies the resulting problem).
- b. This will typically be a negative term (= profit per accepted customer).

| Parameter                   | Value  | Addition to Abbreviation |
|-----------------------------|--|--------------------------|
| <i>Linearity</i>            | <i>Linear</i>  | -                        |
|                             | <i>Non-Linear</i>  | <i>nl</i>                |
| <i>Time dependent costs</i> | <i>Costs can vary between different periods</i>                | -                        |
|                             | <i>Equal over all periods</i>                                  | =                        |
| <i>Cost-Constraint</i>      | <i>Costs are unconstrained</i>                                 | -                        |
|                             | <i>Budget constraint for costs of this type<sup>a</sup></i>    | <i>budg</i>              |
|                             | <i>Technical constraint for costs of this type<sup>a</sup></i> | <i>tech</i>              |

Table 6: Cost Model Additions

- a. See Section 3.2.2 for an example.

Linear fixed and variable allocation costs are described by “FV” while “ $F = V_{nl}$ ” would denote linear fixed costs that are the same in all periods and non-linear variable costs.

### 2.1.5 Intermediary

Note that the intermediary is the component where solution techniques towards MPRASE problems are conceptually located. Therefore, it captures the target function of the optimization problem.

| Parameter                          | Value  | Abbreviation        |
|------------------------------------|--|---------------------|
| <i>Part of the Target Function</i> | <i>All Cost Terms of Cost Model</i>            | *                   |
|                                    | <i>Individual Cost Terms of the Cost Model</i> | <i>F,V,U,R,C...</i> |

Table 7: Intermediary Model

If all cost terms of the cost model are to be optimized (minimized) this is indicated by “\*”. If the cost model is “FV” but the intermediary only “V” this means that only the variable costs are to be minimized. This usually makes only sense if the other cost term (“F”) is under a constraint of any type (e.g. “ $F_{budg}$ ”, see 3.2.2 for an example).

### 2.1.6 Edge

The edge model encompasses the nature of knowledge about capacity demands at the system edges, i.e., whether deterministic or statistical knowledge about future demands is available or if total uncertainty needs to be assumed. .

| Parameter                            | Value                                | Abbreviation        |
|--------------------------------------|--------------------------------------|---------------------|
| <i>Parameter</i>                     | <i>All</i>                           | *                   |
|                                      | <i>Cost Term</i>                     | <i>F,V,U,R,C...</i> |
|                                      | <i>Demand</i>                        | <i>D</i>            |
|                                      | <i>Budget / Technical Constraint</i> | <i>Budg / Tech</i>  |
|                                      | <i>Provider's capacity</i>           | <i>Cap</i>          |
|                                      | ...                                  |                     |
| <i>Knowledge about Future Demand</i> | <i>Deterministic</i>                 | -                   |
|                                      | <i>Stochastic</i>                    | <i>S</i>            |
|                                      | <i>Discrete stochastic</i>           | <i>D</i>            |
|                                      | <i>Total Uncertainty</i>             | <i>T</i>            |

Table 8: Edge Model

### 2.1.7 Taxonomy for the Complete Problem Incarnation

We can now describe each MPRASE problem incarnation by describing all of the six components as follows:

*Customer / Provider / Resource / Cost / Intermediary / Edge*

“1 | 1 | 1 | FV | \* | \*” thus describes the MPRASE problem incarnation with one customer, one provider, a one-dimensional resource, linear fixed and variable costs that are to be minimized under deterministic knowledge.

### 2.1.8 Is the taxonomy complete?

As the MPRASE approach is still new and we are still investigating many of its aspects we cannot be sure that this taxonomy is really complete.

## 2.2 MPRASE Solution Strategies

The solution strategies can also be divided into several classes. First they can be exact or heuristic, while heuristics can be further divided into metaheuristics (like genetic algorithms or tabu search) and construction and improvement technics.

A strategy can look forward into the future in order to anticipate future development (look-ahead) or can use only input data for the actual period (myopic). If a strategy is adapting its behaviour in time by looking at past decisions it is called adaptiv.

Stochastic strategies can come to different solutions for the same problem instance if run twice while deterministic strategies will always come to the same solution for one specific problem instance.

### 3 MPRASE Problem Incarnations

#### 3.1 Overview

Table 9 gives an overview of the MPRASE problems modelled and treated in varying detail in this and other papers. It illustrates our basic goal of treating MPRASE problems in an integrated fashion by making their relations explicit and using that knowledge for solution approaches.

The first row shows a nice human readable name for the problem incarnation. The problem incarnations are then identified uniquely with the taxonomy entry. We also show again the individual components, differences to the simplest model - the Single Provider Problem “1 | 1 | 1 | FV | \* | \*” - are marked in bold.

| Problem Incarnation  | Taxonomy                                | Customer                             | Provider                                       | Resource               | Cost  | Inter-mediary  | Edge                    | Model in              | Solved in             |
|--|---|--------------------------------------|--|------------------------|---|--|-------------------------|-----------------------|-----------------------|
| <i>Single provider problem (SPP)</i>                                   | $1   1   1   FV   *   *$                | <i>single customer</i>               | <i>single provider</i>                         | <i>one-dimensional</i> | <i>linear fixed and variable allocation costs</i>                                       | <i>all cost terms</i>  | <i>deterministic</i>    | [1]                   | [1]                   |
| <i>Flexible demand problem with admission control</i>                  | $N_{AC}   1   1   FVC   *   *$          | <i>multiple rejectable customers</i> | <i>single provider</i>                         | <i>one-dimensional</i> | <i>linear fixed and variable allocation costs, profit per accepted customer</i>         | <i>all cost terms</i>  | <i>deterministic</i>    | [1]                   |                       |
| <i>Flexible demand problem with degraded quality</i>                   | $I_{DQ}   1   1   FVR   *   *$          | <i>single degradable customer</i>    | <i>single provider</i>                         | <i>one-dimensional</i> | <i>linear fixed and variable allocation costs, penalty costs for unsatisfied demand</i> | <i>all cost terms</i>  | <i>deterministic</i>    | [1]                   |                       |
| <i>Multi-provider problem (MPP)</i>                                    | $1   N   1   FV   *   *$                | <i>single customer</i>               | <i>multiple providers</i>                      | <i>one-dimensional</i> | <i>linear fixed and variable allocation costs</i>                                       | <i>all cost terms</i>  | <i>deterministic</i>    | [1]                   | [1]                   |
| <i>capacitated MPP</i>   | $1   N_{Cap}   1   FV   *   *$          | <i>single customer</i>               | <i>multiple providers with finite capacity</i> | <i>one-dimensional</i> | <i>linear fixed and variable allocation costs</i>                                       | <i>all cost terms</i>  | <i>deterministic</i>    | [1] <sup>a</sup>      | [1]                   |
| <i>Token Bucket Allocation Problem (TBAP)</i>                          | $1   1   N^{TB}   FV   *   *$           | <i>single customer</i>               | <i>single provider</i>                         | <b>Token Bucket</b>    | <i>linear fixed and variable allocation costs</i>                                       | <i>all cost terms</i>  | <i>deterministic</i>    | [1] <sup>a</sup>      |                       |
| <i>Simplified TBAP</i>   | $1   1   N^{TB}   F_{\infty} V   *   *$ | <i>single customer</i>               | <i>single provider</i>                         | <b>Token Bucket</b>    | <b><i>infinite fixed and linear variable allocation costs</i></b>                       | <i>all cost terms</i>  | <i>deterministic</i>    | [1] <sup>a</sup>      |                       |
| <i>Limited number of allocations problem with technical constraint</i> | $1   1   1   F_{budg} V   V   *$        | <i>single customer</i>               | <i>single provider</i>                         | <i>one-dimensional</i> | <i>linear fixed and variable allocation costs</i>                                       | <b><i>minimize only variable costs, fixed costs under budget constraint</i></b>    | <i>deterministic</i>    | <i>here</i>           |                       |
| <i>Limited number of allocations problem with budget constraint</i>    | $1   1   1   F_{tech} V   V   *$        | <i>single customer</i>               | <i>single provider</i>                         | <i>one-dimensional</i> | <i>linear fixed and variable allocation costs</i>                                       | <b><i>minimize only variable costs, fixed costs under technical constraint</i></b> | <i>deterministic</i>    | <i>here</i>           |                       |
| <i>Uncertain SPP</i>   | $1   1   1   FV   *   D_T$              | <i>single customer</i>               | <i>single provider</i>                         | <i>one-dimensional</i> | <i>linear fixed and variable allocation costs</i>                                       | <i>all cost terms</i>  | <b><i>uncertain</i></b> | [2], [3] <sup>a</sup> | [2], [3] <sup>a</sup> |

Table 9: MPRASE problem incarnations

a. work in progress

### 3.2 Example Problem Incarnations

In order to give some examples for the taxonomy usage we now present the basic and most simple problem of all, the SPP and some problem incarnations that have not been discussed so far as MIP [9] problems.

#### 3.2.1 Single Provider Allocation Problem

Let us first look at the single provider allocation problem (SPP). The customer has capacity demands  $b_t$  that must be fully satisfied at every discrete time interval  $t = 1, \dots, T$ . As the edge model is deterministic, the demand is known in advance for all periods. Capacity is requested from a single provider who is charging a fixed costs  $f_t$  for every allocation and variable allocation costs  $c_t$  per reserved capacity unit and period. A new allocation is constituted by a change in the allocated capacity. Allocated capacity is available in the period the allocation is made and in all subsequent periods until the next allocation is made. Note that the allocated and not the really used capacity causes the costs. Using two types of variables and a number of parameters, this problem can be formulated as model M1.

##### M1 Single Provider Problem - SPP

Variables:

$x_t$  Amount of reserved capacity in period  $t = 1, \dots, T$ .

$z_t$  Binary variable, 1 if a allocation is made at beginning of period  $t = 1, \dots, T$  and 0 otherwise.

Parameters:

$b_t$  Demanded capacity in period  $t = 1, \dots, T$ . Demand is assumed to be greater than 0.

$f_t$  Fixed allocation costs, costs per allocation. We assume positive costs ( $f_t > 0$ ).

$c_t$  Variable allocation costs, costs per reserved capacity unit per period.

$x_0$  Allocation level before the beginning of the first period.

$M$   $M$  is a sufficiently high number (e.g.,  $\max \{b_t\}$ ).

$$\text{Minimize} \quad \sum_{t=1}^T f_t z_t + \sum_{t=1}^T c_t x_t \quad (1)$$

subject to

$$x_t \geq b_t \quad \forall t = 1, \dots, T \quad (2)$$

$$x_t - x_{t-1} \leq M \cdot z_t \quad \forall t = 1, \dots, T \quad (3)$$

$$x_{t-1} - x_t \leq M \cdot z_t \quad \forall t = 1, \dots, T \quad (4)$$

$$z_t \in \{0, 1\} \quad \forall t = 1, \dots, T \quad (5)$$

The objective function (1) minimizes total costs. (2) ensures that demand is fully satisfied in each period. (3) and (4) force  $z_t$  to one whenever  $x_t$  and  $x_{t-1}$  differ, i.e., a new resource allocation takes place. Note that  $z_t$  will be set to 0 in all other cases automatically because of the non-negative entry  $f_t$  in the objective function.

### 3.2.2 Limited Number of Allocations Problems

We now look at two problems that differ from the SPP in the way they treat the fixed costs in the intermediary model. We no longer try to minimize the fixed and variable costs, instead we try to minimize only the variable allocation costs (the costs for the amount of reserved resources) and limit the number of reallocations that can be made.

The first problem M2 limits the number of reallocations that can be made with a budget constraint. That means there is only a fixed budget available for the reallocations. If  $f_t$  is set to 1 then B is effectively the number of reallocations that are allowed.

**M2 Limited Number of Allocations Problem with Budget Constraint**

Variables and Parameters: see M1

Variables and Parameters:

B Budget for fixed costs

Minimize 
$$\sum_{t=1}^T c_t x_t \quad (6)$$

subject to (2)-(5)

$$\sum_{t=1}^T f_t z_t \leq B \quad (7)$$

In M3 we model a system where reallocations are technically possible only every  $\Delta T$  periods. The model no longer includes a term for the fixed costs  $f_t$  but still need the variables to measure reallocation ( $z_t$ ).

**M3 Limited Number of Allocations Problem with Technical Constraint**

Variables and Parameters: see M1

Variables and Parameters:

$\Delta T$  Time interval that must pass between two (re)allocations

Minimize (6)

subject to (2)-(5)

$$\sum_{\tau=t}^{t+\Delta T} z_\tau \leq 1 \quad \forall t = 1, \dots, T-\Delta T \quad (8)$$

## 4 Conclusion & Outlook

This paper has described a taxonomy for a so far largely neglected class of network QoS problems related to resource allocation at system edges over multiple time periods. We developed the MPRASE model to classify and describe this class of problems and to analyse their mutual dependencies and relationships. The model consists of the six submodels

describing the individual facets of the different problem incarnations: customer, provider, resource, cost, edge and intermediary. Each submodel can be described by a short abbreviation, the combination of them then identifies the problem incarnation exactly.

The basic model covers a huge amount of interesting problem incarnations. Many of them are discussed and solved in an integrated fashion in other papers and future works.

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